

# *WebMathematica—based Introductory System of Mathematical Experiments*

**Hitoshi Nishizawa**

Toyota National College of Technology, Japan  
nisizawa@toyota-ct.ac.jp

**Takayoshi Yoshioka**

Toyota National College of Technology, Japan

**With the help of technology like Mathematica, students nowadays have the opportunity to explore the world of mathematics, which is expected to help them to develop mathematical thinking by themselves. But it is not enough for certain students who believe in memorizing rather than thinking when they learn. Such students, surface learners, try to remember everything but avoid to explore or to struggle for getting new ideas from experiments. As the result, they learn less from mathematical experiments, and continue to have difficulty in learning mathematical ideas.**

**We think it necessary for them to build an introductory system that helps them to get into the world of mathematical experiments and to develop their skills of generalization with the help of technology. The skills of generalization here mean finding common patterns from actual experimental results, making conjectures, and confirming them with specific data.**

**We take the transformation of graphs for target domain because it is one of the most influential algebraic ideas for engineering students. Horizontal and vertical shift of graphs, rotation of graphs around horizontal or vertical axis, shrinking and expanding of graphs are the transformations which students are expected to learn. We expect the students to express the changes in graphs, symbolic expressions, and also in written texts.**

**The introductory system consists of two activities. In the first activity: the guided exercise, the student composes a sentence to describe a transformation of an algebraic function, and observes the transformed graph and mathematical expression immediately on his/her screen. The system helps him/her to compose the sentence by providing lists of functions and operations for selection. The student only needs to select them from the lists and input a necessary parameter. The parameter can be a number or a list of numbers. The student observes the transformations in this activity.**

**In the second activity: the independent exercise, a function in algebraic expression and a transformation expressed by a sentence and a graph is given by the system, and the student is to express the transformed function symbolically. He/She is also**

requested to confirm the appropriateness of his/her algebraic expression by substituting a number for the variable of original function, and by observing the transformation of the related point on the graphs. The student makes his/her own decisions and numerical confirmations in this activity.

Through the two-step exercises, the student will have more confidence on the transformation of graphs of mathematic functions, and is ready to enter into the rich world of mathematical experiments.

## 1. Background

Surface learning has been spreading among Japanese high school students in the last decade, even in the learning of mathematics, as the *effective* learning method to pass examinations with the minimum learning effort [1]. Surface learners memorize all the necessary formulas blindly without understanding their embedded mechanism before their examinations, and forget most of them after the examinations. They even try to memorize major solving processes for typical problems if the contents of the examinations are limited. Their answer sheets usually have few written calculating processes, graphical or descriptive explanations. They make curious mistakes in calculations if the teacher requests them to write the calculating processes in detail on their answer sheets. Because they don't have the habit to explain their ideas to others, most surface learners have difficulty in expressing their ideas of solving problems. The authors feel the necessity to guide the surface learners to more meaningful learning of algebra during their first school year in our college.

Technology does not necessarily help surface learners. When they use technology in their learning algebra, they just observe the rapid changing phenomena on their screen as TV programs but are hardly involved in them. They don't make any conjectures, generalize the phenomena, nor confirm the results by actual data. There is a wide gap between their actual learning behavior and the expected use of technology. They need some additional support to learn through technology.

## 2. Feature of Guided Exercise

We tried to prepare such a support as Web-based learning activities for surface learners with the help of Mathematica. The activities guide the learners to more meaningful learning styles and get the learners ready to technology. The first activity, which we call a guided exercise in this paper, uses a sentence written in natural language as a main tool to express the meaning of mathematical transformation. In this paper, it is the transformation of a graph and the related change of symbolic expression of the function. The features of the guided exercise are described in Table 1 in comparison with a virtual experiment and an independent exercise.

A virtual experiment is conducted on a student's computer screen with the direct use of CAS (Computer Algebra System) like Mathematica or DGS (Dynamic Geography Software). CAS helps the student to know the graphical representation of a given symbolically described function dynamically. DGS helps him/her to know the symbolic expression of a given graph when the student modified the graph. In the virtual experiment, we expect the student to observe the transformations of a graph and the related changes of the symbolic expression on the screen carefully, makes his/her own conjecture of the embedded mechanism of the transformations by generalizing the observed phenomena, and confirms the conjecture in the environment by himself/herself. But a surface learner tends to watch the fast changing transformations on the screen like a TV program and makes no positive reflection if there is no interactive communication with the teacher or the other students in class. Virtual experiments only help him/her when such a peer-to-peer communication with the others is provided. Surface learners are not ready to learn through virtual experiments.

Table 1 Features of guided exercise in learning transformation of graphs of mathematical functions

□	Virtual experiment	Guided exercise
Example installation	Direct use of CAS, DGS E - texts on the Web	Step - by - step instruction
Student ' s activity	Observe the transformation in symbolic expression and graphical representation	Compare descriptive expressions to explain the transformation
Positive use	Make conjecture and generalize ideas	Learn the transformation through the description
Negative use	Observe it passively like a TV program	?

In an independent exercise, it is impossible for a student just to watch the screen without making his/her own decision. But a surface learner relies on visible patterns without seeking the embedded mechanism. His/her success is limited in a short term while he/she remembers the patterns. Gradually, he/she comes to make mixture between similar patterns or simply forgets some of the patterns and fails. Then he/she tries random answers, and finally gives up. Independent exercises are too hard for him/her.

A guided exercise is not automatic but not too difficult for surface learners because it uses a sentence written in natural language to describe the graphical transformation at first. Composing a sentence describing a graphical transformation is just like a translation from graphics into natural language. We decide to use a sentence written in natural language to connect the rewriting of mathematical expressions and graphical transformation of the graphs because it helps the learners to confirm what he/she recognizes on the screen externally. Some surface learners understand what is happening on the screen just after reading his/her own explanation.

We have already reported an example of the assistance of sentences in the step-by-step instruction of calculating algebraic fractions [3], where the students learn the mathematical rewriting rules in those sentences. In this paper we show another installation of the approach.

### 3. First Activity: Guided Exercise

In the first activity of the introductory system: guided exercise (Figure 1), the student composes a sentence to describe how he/she transforms the graph of a function and confirms the transformation through the change of the graph and symbolic expression of the function. For composing the sentence, he/she selects a original function to be transformed from the pull-down menu on the screen, selects the kind and direction of transformation from the other menus, inputs the amount of transformation in the form field, and clicks on the execute button on the screen. The transformation is the shrinking/expansion, horizontal/vertical shift, or the rotation of the original graph. The amount of transformation can be given as a number or a list of numbers.

Figure 1 An Example of the guided exercise



Google

# Stretches and Shrinks of Graphs

If you stretch the graph of  $x^2$    times



Stretched graphs of  $x^2$ :  $\{(x/2)^2, (x/4)^2\}$  are

The points, where  $y =$  , on the graphs are

 ページが表示されました

As the executed results, the graphs of original and transformed functions, and the symbolic expressions of transformed functions in not-simplified form are displayed on his/her screen. The not-simplified expression of transformed function, for example,

$$y = \left(\frac{x}{2}\right)^2$$

is more informative in revealing the embedded mechanism of the transformation than the simplified expression

$$y = \frac{x^2}{4}$$

In this case, the mechanism of two times horizontal expansion of the graph of easier to recognize in

$$y = \left(\frac{x}{2}\right)^2$$

than in

$$y = \frac{x^2}{4}$$

When the amount of transformation is given as a list of numbers, a series of transformed graphs are plotted simultaneously in different colors. Although this activity is similar to the direct use of `Plot[]` function of Mathematica, the composed sentence describing the transformation and the not-simplified symbolic expression of transformed function gives valuable assistances to some students.

After observing the transformed graphs on the screen, the student also confirms the transformation as the position-change of a typical point on the graphs, which also assists the description in the sentence. We also expect it to motivate the student to have the habit of confirming his/her answer with actual data.

#### **4. Second Activity: Independent Exercise**

In the second activity: independent exercise, the student confirms his/her understanding about the transformation of graphs. In the exercise, a symbolic expression of a mathematical function to be transformed, a sentence describing how to transform the graph of the given function, and a pair of graphs; the original and transformed graphs, are given by the system on the screen. The student inputs the symbolic expression of the transformed function into the form field and clicks on the execute button. The system plots the graphs of the input and original functions and displays them just under the given graphs. So the student can compare two pairs of the graphs.

He/She can also confirm the correctness of his/her input expression by substituting a typical number for the variable of original function, and by observing the transformation of the related point on the graphs. The student can test his/her understanding about the transformation and confirm numerically in this activity.

If a student find it difficult to achieve the independent exercise for a certain function, it is possible to move back to the guided exercise with the function as the default target function in the guided exercise. In this case, the guided exercise becomes a hint page for the independent exercise page.

#### **Figure 2 An Example of the independent exercise**

Guided Exercise – Microsoft Internet Explorer

ファイル(F) 編集(E) 表示(V) お気に入り(A) ツール(T) ヘルプ(H)

戻る 検索 お気に入り

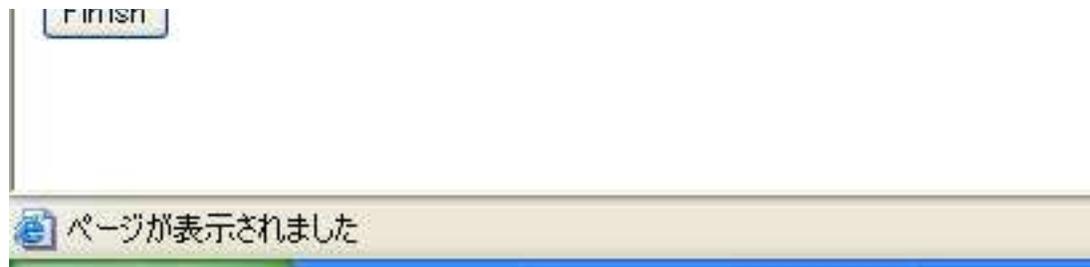
アドレス(D) <http://carrot.ee.toyota-ct.ac.jp/webMathematica/MSP/HyperMe>

Google 検索 4 をブロック

# Translations of Graphs

If you translate the graph of  $y = \sin[x]$  horizontally 2 units to the right,

the translated function is



## 5. Installation of the Exercise Pages

The exercise pages are served as Web-pages dynamically created by WebMathematica, with the help of symbolic calculating ability of Mathematica, the teaching strategies described in custom functions written in Mathematica language, and the learning histories of students stored in the database. Students get access to the exercise pages through the campus LAN or the Internet from their own computer terminals connected to the network. Because the exercises are installed in our learning environment [4] for the students, they don't need to learn new rules to use them. It is also possible to invite selected students to the exercise pages for their extra work based on their learning histories at the collaborated off-line exercises because the learning histories in the exercises are stored in the database of the learning environment. In such a way, the introductory system can be used in remedial lessons for slow learners.

## 6. Discussion

We decided to prepare the introductory system so as to guide the gradually increasing number of surface learners among fresh students of our college. Because their learning style has been build up in a long term during their junior high school days and they believe in the *effectiveness* of it, it takes rather long time and effort for the teachers to guide them into more meaningful learning style. In the past, we had failed to change the learning style of some of them through a series of paper-and-pencil exercises, where the teacher had advised them to write more explanations on their answer sheets and had pointed typical curious mistakes caused by surface learning. As a result, the students suffered poor performance in the engineering subjects in their later school years. The failure partly caused by the time-separated feedback, which would be improved by the used of this system.

The effectiveness of the system will be confirmed by observing the richness of descriptive and graphical explanations in their answer sheets later in this school year.

## 7. Conclusion

We have built an introductory system that helps surface learners to get into the world of mathematical experiments and to learn with the help of technology. The introductory system possesses two activities; a guided exercise and an independent exercise of transforming the graphs of given mathematical functions. In the exercises, description in a sentence of natural language plays the key role, and the confirmation with numerical data assists it to change the learning style of the students. The system uses the symbolic processing power of Mathematica through the Web-connecting function WebMathematica and is available through computer networks.

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## **References**

- [1] S. Fujisawa, Surface Learning (in Japanese), Shin-yo-sha (2002).
- [2] H. Nishizawa, Interactive On-line Exercises of Basic Mathematical Functions, Proc. of IMS99 (1999).
- [3] T. Yoshioka et al, Step-by-step instruction of symbolic calculation, Proc. of ATCM00, pp186-192 (2000).
- [4] H. Nishizawa et al, An implementation of a hypermedia learning environment for a small group, Trans. of Japanese Society for Information and Systems in Education, 15(4) pp249-253 (1999).