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Mathematics with Power

Proceedings

of the

Third International *Mathematica* Symposium

***IMS* '99**

Editors:
V. Buber, A. Hietamäki, V. Keränen

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PREFACE

Welcome!

At RISC, we enjoy organizing conferences and welcoming our friends from around the world who share with us the interest and enthusiasm for algorithmic mathematics and, in particular, symbolic and algebraic computation.

I am especially happy to host *IMS* ’99 at RISC, not only because of the importance of *Mathematica* for current mathematics and its applications but also because of my personal interest in this system: When *Mathematica* 3.0 came out, I immediately felt that this is, finally, a system in which I could program, in a natural and easy way, the type of 'natural style' automated and interactive theorem provers which I had in mind for a long time. Thus, I decided to base the development of our Theorema system on *Mathematica* and I am finding that we gain a lot of speed, convenience, and practicability from this decision.

The future of mathematics will change drastically by systems like *Mathematica* although, amazingly, this is still not recognized by the majority of mathematicians. The point is that these systems do not only help in solving practical mathematical problems and carrying out 'mathematical experiments' but inventing mathematical algorithms and designing mathematical software systems give rise to challenging new mathematical problems that can only be solved by developing new mathematical theories and non-trivial new mathematical results. In fact, algorithmic solutions to problems in an area of mathematics tend to need 'deeper' mathematical results and, correspondingly, more difficult and sophisticated proofs than the non-algorithmic exploration of the area. Thus, the feedback cycle between mathematical research and mathematical systems development will generate a new and vigorous push to mathematics unparalleled in history.

I thank the *IMS* secretariat for having chosen RISC as the site of this conference and I wish all participants a fruitful and enjoyable interaction. And, please, come back!

Bruno Buchberger
Chair of Organising Committee
Head of RISC; Professor, Johannes Kepler University, Hagenberg, Austria

IMS ’99 is the third incarnation of a biennial computer algebra conference. The previous conferences took place in Southampton, England, and Rovaniemi, Finland. As did the previous symposia, *IMS* ’99 has attracted researchers, educators, engineers, and in general people from many walks of life who all share an interest in *Mathematica* and symbolic computation. We should expect to see many new and interesting applications of symbolic computation in education, scientific computing, and industrial applications. We are looking forward to a lively exchange of ideas within and also between the various groups of attendees.

Mathematica, the software artifact that has brought us all together, is now in its fourth release. Those who still remember the first version continue to be amazed at the rapid pace of innovation and progress. Of course, there is the burden of additional complexity that the user has to deal with, and we hope to learn more about the recent release from several WRI members who will give tutorials.

Veikko Keränen and Antero Hietamäki, together with Vladimir Buber, have again graciously agreed to deal with the conference proceedings. They also dealt admirably with the tedious task of refereeing the submissions. However, in a break from the previous conferences a decision was made to combat the automatic obsolescence of the printed medium by placing all documents on the Web. We trust that web-publishing will greatly enhance the dissemination of the material, and make it accessible to a much wider audience. We also strongly encourage authors to take full advantage of the new medium and provide updates, fixes, new applications and such like.

We are greatly indebted to RISC and in particular Bruno Buchberger for hospitality and support. RISC has been at the forefront of symbolic computation research as well as application for many years. The unique RISC combination of a historical environment and a state-of-the-art research facility, which in addition maintains close ties to industry, will provide an excellent background for the conference. It is difficult to imagine a more perfect environment for *IMS*.

As Gautam Dasgupta, who has again made invaluable contributions, pointed out in the *IMS* ’97 proceedings: The sun will not set over *IMS*. As we have since moved away from the Arctic Circle, this is not quite literally true any more, but in spirit Gautam's maxim still holds.

Klaus Sutner
Co-Chair of Organising Committee
Professor, Carnegie-Mellon University, Pittsburgh, USA

The editorial work for *IMS* ’99 was carried out at the School of Technology, Rovaniemi Polytechnic, Finland, in cooperation with the Murmansk State Technical University, Russia. The support of these two institutes is gratefully acknowledged.

The editors received over 60 abstracts, out of which 40 final papers are included in the proceedings. The fact that we received papers from five continents makes the *IMS* truly an international event.

We wish to thank the members of the Programme Committee for reviewing the abstracts and the useful advice they provided for the authors. We would also like to thank all those who submitted abstracts, and, of course, all the final participants for their contributions.

The people who have, over the years, taken the efforts to learn to master the wide-ranging features of technical computing systems, are now able to use the power at hand in most creative ways. This was clearly seen in the papers and made the editors’ job a rewarding experience.

We decided to publish the full conference proceedings on the web which made it possible for the authors to make last-minute modifications and allowed them to develop their works much further than was possible before. We have long been feeling that this kind of profound way of working is characteristic for *IMS* participants - and now the quality really emerged.

We believe that this way of publishing spreads the contents of the conference to a much wider audience than before. The electronic format is also convenient for the readers because the hyperlinks in the electronic papers make accessing reference information fast and easy. Moreover, the interactive *Mathematica* notebooks make instant experimentation possible for every *Mathematica* user.

The complete material can be found in http://algebra.rotol.ramk.fi/ and in http://www.risc.uni-linz.ac.at/conferences/summer99/ims99.

The schedule of the symposium contains paper numbers. These were assigned chronologically during the refereeing process.

Cooperation with the local organisers at RISC has been an essential part of publishing the papers and the schedule on the web. We take this opportunity to express our gratitude to Bruno Buchberger and Claudio Dupre for their active role in placing material and hyperlinks on the RISC home pages, and for promptly answering our numerous questions related to the practical arrangements concerning the conference presentations of the reviewed papers.

Vladimir Buber, Antero Hietamäki, and Veikko Keränen
Editors

Right after the first International *Mathematica* Symposium in Southampton, England, Bruno Buchberger volunteered to hold *IMS* '99 at this prestigious institute. Since the second symposium in Rovaniemi, Finland, Bruno and his colleagues from RISC, notably Claudio Dupre have been traversing the globe via email to organize this third meeting. Veikko Keränen from Finland and Klaus Sutner from the US have thoroughly examined all possibilities the world wide web offers to make this symposium a success.

As anticipated, the quality of papers employing computer algebra formulations have steadily been rising. In the meantime the software, *Mathematica*, has entered into its fourth version with added dimensions both in the frontend and kernel. Professional teachers from Wolfram Research will hold special tutorials to initiate this body of users to the modern features for international communication and exchange of mathematical ideas and results. Thanks for their efforts in structuring the MathML standard.

The title of the proceedings, Mathematics with Power, correctly reflects Antero Hietämaki's computing experience as a physicist. I can say without any hesitation that all of us in the executive committee feel that the real strength of computing is derived from its mathematical foundation.

In the on-going arena of mathematics education members of *IMS* conducted almost a week long program in Thessaloniki, Greece, under the auspices of the International Society of Computer Music. This was a follow up of the extensive High School teaching during *IMS* '97 in Finland. Currently serious negotiations with international funding agencies are taking place for producing TV programs to teach mathematics. *IMS* and the participants in the symposia are to be credited for these ingenious developments.

I must say that we are falling behind as with respect to *Mathematica* training of college professors and especially the teaching assistants/tutors, who would deal with undergraduates. Also, sadly enough, the globalisation of economy has not made it easier for the East European, Asian, African and Latin American countries to procure this powerful software. An affordable and just pricing structure is urgently warranted from the makers of *Mathematica.*

As an engineer it is a delight to see that the community is integrating *Mathematica* in design oriented calculations. Especially the version 4 enhancement of numerics should make the tasks more enjoyable. In many situations the roadmap for a C++ code is currently drawn on the *Mathematica* notebook workbench. We are working on popularising applications of computer mathematics in experimental studies of data acquisition from the real world engineering problems.

In *IMS* '95, Peter Mitic introduced Jerry Connor of MIT to illustrate engineering design tools. In *IMS* '97 Perry Cook of Princeton focused on sound and music, and in this symposium Jacques Treil, who is working closely with the Medical center in Toulouse, France, will deliver a keynote on medical application of computer mathematics in maxillo-facial surgery.

Now the challenge is to attract liberal art colleagues in History and Law. I understand that the sister organisation, the International Arctic Seminar - IAS, is actively exploring such possibilities under the leadership of Oleg Klimov of Pedagogy and Vladimir Sereda of Mathematics and Computer Science, Murmansk, Russia, with Veikko Keränen and Vladimir Buber of the *IMS* editorial board. Robert Kragler, Physics, Weingarten, and Jouko Teeriaho, Mathematics, Rovaniemi Polytechnic, and Ronald Russay, Chemistry, UC Berkeley, have already approached the appropriate funding agencies.

The state of the *IMS* is healthy and strong, so let us get back to work to make mathematics enjoyable and useful for all the citizens of the world.

Gautam Dasgupta
Chair of executive committee,
Professor, Columbia University, New York, USA

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#5
R. Knapp
Numerical Approximations of the Nonliner Schrodinger Equation with *Mathematica*

#7
A. Wrangsjö, P. Fritzson, K. Sheshadri
Transforming Systems of PDEs for Efficient Numerical Solution (123202 .pdf)
Transformations.nb (92215)

#9A
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An Algorithm Computing Homology Groups of Semidirect Products of Finite Groups

#9B (poster)
V. Alvarez, M.D. Frau, P. Real, B. Silva
An Algorithm Computing Homology Groups of Commutative Differential Graded Algebras with Linear Differential

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Non-trivial Asymptotic Formulas by Symbolic Computation (167458 .nb)

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CFLP: a *Mathematica* Implementation of a Distributed Constraint Solving System (164591 .pdf)

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Dynamic Optimization and Differential Games with Applications to Economics (158202 .pdf)
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The Application of Winding Numbers to the Interior Problem of Nonconvex Polygons and Polyhedra (662780 .nb)

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Mathematica-based Package for Studying Ordinary Differential Equations and for Analyzing the Learning Process
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An Example of Computer-based Study Material in Mathematics: Planning Principles and Realization
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Computing of Curvature Invariants in Arbitrary Dimension (138601 .pdf)

#54
T. de Alwis
Normal Lines Drawn to Ellipses and Elliptic Integrals (531637 .pdf)

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Musical Representations of the Fibonacci String and Proteins Using *Mathematica*

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G. Dasgupta
System Stochasticity: Discrete Formulation with *Mathematica*

Abstracts

Ordered by Paper Number

#2
"You're Doing Simulations with Your Students, so Why Are You Using *Mathematica*?"

Phil Ramsden
Department of Mathematics, Imperial College of Science, Technology and Medicine, London, UK

Abstract
At Imperial College, staff from the Mathematics and Chemistry departments have set up computer-based "Mathematics Laboratories" for first year Chemistry undergraduates. We´ve provided several types of activity in these laboratories, but one of the most important kinds involves students? setting up and studying models and simulations of chemical conditions and processes. Dedicated simulation software exists for this sort of thing, much of it offering a lot of presentational sophistication and dynamic interactivity. But actually, all our simulations are implemented entirely in *Mathematica*; moreover, our presentation style is simple and stark in the extreme. *Mathematica* works for us as simulation and modelling software because it allows us to be open in our approach to design. All the code that is used to set up and run the simulations is visible to the students, and indeed we require them to engage with it directly. The relationships between the underlying mathematics, the chemistry that is being modelled, and the computer code that implements the model can be explicit rather than hidden. Students can be encouraged to adopt a critical attitude towards mathematical models and to develop their own ideas and approaches. On the other hand, *Mathematica*´s power and range enable us to set up fairly realistic and uncontrived problems, and to place some mathematical elements of these problems firmly in the background if we choose. I illustrate these strengths of *Mathematica* for our purposes using the example of a series of assessed problems. These concern the forces between ions (modelled as Newtonian point charges) and the chemical dynamics to which these forces give rise. I show examples of closed and open-ended tasks, and present excerpts of students? work on these tasks.

#3
Exploring the Theory of Geometric Bifurcation

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Abstract
This paper will deal with the Theory of Geometric Bifurcation which the author developed in 1986. The theory developed in those papers is very general and abstract. So, until a symbolic software program such as *Mathematica* came along, it was very difficult to examine concrete examples which would illustrate and explain the theory. In this paper we will look at examples of bifurcating branches of solutions of nonlinear: algebraic equations, ordinary differential equations, and partial differential equations.

#5
Numerical Approximations of the Nonliner Schrodinger Equation with Mathematica

Robert Knapp
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In the course of investigating of the effect of a random potential on pulses with the cubic nonlinear Schrödinger equation,

$$i\frac{\partial u}{\partial t}+\frac{\partial^2 u}{\partial x^2}+2\,|u|^2\,u=V(x)\,u$$

I became intrigued by what one should define as the effective width of a pulse or soliton, particularly in cases which are near-integrable. In an attempt to answer that question, I did some numerical simulations with periodic potentials and came across an interesting range of behavior depending on how the length scales of potential and pulse compare. Some of the results are explainable by simple perturbation methods, and some remain more mysterious. I will show you the results I have obtained and conclude with some observations about the effective width.

I will structure this presentation roughly as follows. First, I will give a brief description of the numerical method. Next, I will present a leading order perturbation analysis for a slowly varying potential. Then, I will show how well this theory bears out in numerical simulations with a slowly varying potential. The results start to get more interesting when the assumption that the potential is slowly varying breaks down. Beyond this, some simulations with a rapidly varying potential show some mildly surprising behavior. Finally, I will try to bridge the entire range of length scales with some conclusions and questions.

#7
Transforming Systems of PDEs for Efficient Numerical Solution

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Abstract
A *Mathematica* package to deal with a system of partial differential equations (PDEs) is presented. This package uses explicit finite-difference schemes to handle equations in an arbitrary number of variables that are functions of one spatial variable and time. The code has the flexibility to incorporate any difference approximation specified by the user, and transforms the given system of PDEs into a system of difference equations that can be iteratively solved using the discretized forms of initial and boundary conditions. The iteration is made considerably faster by converting the *Mathematica* code into an optimized C++ code using the MathCode C++ compiler[1]. Examples are presented in which the generated C++ code runs about a thousand times faster than the *Mathematica* code.

#9A
An Algorithm Computing Homology Groups of Semidirect Products of Finite Groups

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In this paper we give an implementation in *Mathematica* of an algorithm for computing homology groups of semidirect products of groups of type (Z/Zm x Z/Zn, j) where j is an action of the group Z_n on a group Z_m. This algorithm has been extracted from [``Small free resolutions for semidirect products of groups'', V. Alvarez, J. A. Armario and P. Real. Communication presented in the International Colloquium on Topology, (Gyula, Hungary, August 9-15, 1998)].

#9B (poster)
An Algorithm Computing Homology Groups of Commutative Differential Graded Algebras with Linear Differential

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We present here an implementation in *Mathematica* of an algorithm for computing the integer algebra homology of commutative differential graded algebras with linear differential. This algorithm is essentially based on the method described in [``HPT and computability of the homology of commutative DGA-algebras''. V.Alvarez, J.A. Armario, P. Real and B. Silva. Conference on Secondary Calculus and Cohomological Physics. Moscow, August,1997].

#10
Code Generation for Simulation and Control Applications

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Abstract
The use of *Mathematica* in combination with MathCode C++ is illustrated in a context of modeling of dynamical systems and design of controllers. The symbolic tools are used to derive a set of nonlinear differential equations using Euler-Lagrange equations of motion. The model is converted to C++ using MathCode C++, which produces an efficient implementation of the large expressions used in the model. The exported code is used for simulations, which illustrates that *Mathematica* in combination with MathCode C++ can be used to do accurate and powerful simulations of nonlinear systems. Controller synthesis is performed where the resulting controller is exported to C++ and run externally. The applications presented are a seesaw/pendulum process and aerodynamics of a fighter aircraft.

#12
MathModelica
– a new modeling and simulation environment for *Mathematica*

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Abstract
MathModelica is a *Mathematica* extension, which provides a modeling, and simulation environment for *Mathematica* based on the new standard of physical modeling languages called Modelica. Modelica is a new object-oriented multi-domain modeling language based on algebraic and differential equations. In this paper we present a language and an environment, MathModelica, that integrates different phases of the Modelica development lifecycle. This is achieved by using the *Mathematica* environment and its structured documents, “notebooks”. Simulation models are represented in the form of structured documents, which integrate source code, documentation and code transformation specifications, as well as providing control over simulation and result visualization.

Import and export of Modelica code between internal structured and external textual representation is supported. *Mathematica* is an interpreted language, which is suitable as a scripting language for controlling simulation and visualization. *Mathematica* also supports symbolic transformations on equations and algebraic expressions which is useful in building mathematical models.

#13
Refined Constitutive Shell Equations with *MathTensor*

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April 27, 1999
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Abstract
The paper presents the results of research on refinement of constitutive shell equations with the MathTensor package of the *Mathematica* system. The obtained results satisfy the last equation of equilibrium and better describe the shell behaviour.

#15
Symbolic Computations in Problems of Mechanics

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Abstract
In the report long-term experience of the authors on development of algorithms and automation of a research of complicated mechanical and controlled systems is considered. We have created specialized systems of computer algebra (CAS) and software packages "Dynamics", "Mechanic" possibility and the algorithms of which are described in [1-5 and other]. Currently we are developing software system [6-9] for solving mechanical problems based on CAS "*Mathematica*" [10]. The software allows to automatize, and consequently, essentially to speed up processes of modelling and dynamic analysis of complicated systems, to avoid errors at all stages of researches. The base of the algorithms, which are realized in these packages, was formed by classical methods of analytical mechanics and stability theory. Our experience with symbolic computation packages allows us to conclude that CAS are perspective tool for researches in the eld of theoretical mechanics.

#16
PLEMATH SYSTEM: Linear and Integer Linear Programming in *Mathematica* System

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Abstract
The PLEMATH System has been built as an educational tool for linear Programming and Integer Linear Programming. In this work we show examples that illustrate the use of the system. We describe how the commands executing the di erent algorithms incorporated to PLEMATH have been made, as well as their possibilities and limitations. Moreover, we exhibit some ideas for further extensions of this work. No knowledge of *Mathematica* is supposed.

Keywords: Linear Programming, Integer LinearProgramming, *Mathematica*

#17 A&B
Automata Containing Evolutionary Algorithms: Behavior and Learning Under Law

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Abstract
To date, the study of cellular automata and genetic programming have proceeded largely on parallel paths. This article studies the evolution of cellular automata in which the cells each contain data and genetic programs. The immediate application of this convergence is to study how different legal rules affect the evolution of learning and behaviour in an economy. The typical "Coarse Theorem" scenario in which neighbours engage in activities that may detrimentally affect each other is used for this study [Chandler, 1997].

Using the *Mathematica* programming language, the authors create a ring of sites each of which contain genetic programs for determining the behaviour and learning strategy of that site. The genetic programs

are created using a flexible template mechanism that facilitates specification of a complex grammar for permissible programs (as was already demonstrated for the evolution of Lindenmayer systems and plant growth programs [Jacob, 1995]). The rich set of primitives of these programs include sensing operations such as determination of the behaviours of neighbouring sites and mating operations that permit the programs of neighbouring sites to be adopted. These primitives thus permit the sites to learn from each other and, recursively, to learn how the other sites themselves learn. This is a novel approach of combining evolutionary algorithms and, in particular, genetic programming techniques [Jacob, 1997a, 1997b], with interacting agents modeled by cellular automata.

The ring of sites (the economy) evolves through a multi-stage iteration. First, the programs of the sites are evaluated to determine the behaviours at the various sites. The behaviours are in turn evaluated to determine certain global parameters (such as the prices of various commodities within the economy) and the consequences of the local interactions created as a result of the behaviours and the legal rules applicable thereto. This process permits a computation of the fitness of each of the sites within the economy. The final state of each evolutionary iteration permits the programs at each site to mutate based in part on the fitness of the site. Evolution is modelled through the repetition of these iterations.

The article then examines how different rules of law regarding the interaction of the sites alters both the evolution of learning and the development of the economy.

#18
Interactive On-line Exercises of Basic *Mathematica*l Functions

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Abstract
In this paper, a system is described for serving on-line exercises of Mathematical functions for high-school students. The system basically depends on WWW technology and uses WWW browsers running on students' computers for displaying questions, answers, and explanations. An exercise page has a graph of a mathematical function and requests a student to fill the text field with a mathematical expression appropriate for the graph. When the student clicks on the "evaluate" button, the expression is sent to the system server and compared with the answer expression. If the expression is not equal to the answer, graphs of both expressions, and the comment on how different his expression to the answer are displayed. Comparing the graphs and reading the comment, the student is able to continue his guessing work until he gets the correct expression. The exercises have been developed for students who have difficulty in understanding mathematical functions in the author´s algebra course, who seem to lack graphical images of the functions they learned in the classes. They usually try to remember the written rules for converting a mathematical function to its graph and reverting it. Such attempts sometimes succeed in simple cases like linear functions but fails in quadratic functions or more complicated functions because the rules for converting those functions become too complicated just to remember. What they lack is rich experiences of handling real graphs. Although recent development of electrical worksheets and symbolic computing program like *Mathematica* made it easy and quick for a student to draw graphs, it is only half the way to the understanding. Because the drawing is automated and includes usually few guessing works, the student does not necessarily have ideas about, for example, the effect of a coefficient value on the shape or the position of a graph. The only rigid knowledge he has is how he operates the program. The on-line exercises provide with the other half. The first thing a student does in the exercise is a guess which tries to describe the given graph correctly. There is no instructions of expressing the graph nor hints before the first guess. After the first guess is done, then comes the hint or comment which help him to make the next guess. This makes a kind of experiment on the desktop. A series of the experiments tell him the behaviour of graphs in detail and the relation with the expressions. Adding to that, because the evaluation are done symbolically using *Mathematica* functions, wide variety of expressions are allowed, which is easier to accept for usual students. The exercise provides the students with opportunities to learn from their mistakes and to build their own theory about mathematical functions.

#19
Integration of *Mathematica* into an Information System Supporting Chemical Process Development

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Abstract
Mathematica has been used as an environment for object modelling of a new chemical pulping process. The usability of *Mathematica* has been enhanced by integrating it into an Information System (IS) supporting chemical process development. The implementation of IS is based on a new phenomenon driven process design methodology. The integration into the IS results in an easy way to change and give process structures for *Mathematica* and makes *Mathematica* an attractive alternative compared to commercial simulators.

#20
Mathematica and Didactical Innovation

G. Albano, A. Cavalloney, C. D'Apice, S. Salerno

Abstract
A package for the education in mathematics focuses on the study of a conic is presented. The package is structured in an hypertextual way, including a part of theory and a guided lab for the practical session. The most meaningful innovation is given by the verification routines. The user may solve exercises by hand and check the solutions he/she finds using two verification functions.

#21
Non-trivial Asymptotic Formulas by Symbolic Computation

Giuliano Gargiulo
Saverio Salerno

Abstract
We want to study the asymptotic behaviour of the complete elliptic integral of the first kind K(m) when m->1. This is motivated, for example, by the occurrence of K(m) as capacitance of a circular capacitor with slit (or similar geometries - see e.g. [9]) -, m being essentially the ratio between the slit's length and the radius of the circle. We show that the analysis of the asymptotic behaviour can be done in several ways (including series expansion and summation, symbolic integration and computation of limits) using both the numerical and symbolical capabilities of state-of-the-art Computer Algebra Systems.

#22
CFLP: a *Mathematica* Implementation of a Distributed Constraint System

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Abstract
The need for combining different solvers into a system that is able to process constraints that can not be solved by a single solver is widely recognized. Of particular interest is the design of a distributed system based on a flexible architecture that supports an easy integration and cooperation of new constraint solvers.

CFLP (Constraint Functional Logic Programming system) is a distributed software system consisting of a functional logic interpreter running on one machine and various specialized constraint solvers that may run on different machines in a network environment. A scheduler running as a separate process serves to coordinate the constraint solving processes.

The system is fully implemented in *Mathematica* 3.0 as a collection of packages which extend the solving capabilities of a normal *Mathematica* session. The functional logic interpreter is based on an extension of the calculi LNC (Lazy Narrowing Calculus) and Higher-Order LNC, which are known to be sound and complete for many equational theories of practical interest. Since *Mathematica* is based on a higher order term rewriting evaluation strategy, the implementation of the calculus is straightforwardly supported. The calculus was extended in two main directions:

* special inference rules for handling equational constraints over various domains
were added,
* the capability to specify AND and OR parallelism.

A computational model for solving these constraints in a distributed manner was designed and implemented. The communication among the system components is based on the MathLink library.

The constraint solvers integrated in the current implementation are implemented on top of the constraint solving libraries provided with *Mathematica*. It provides an easy-to-use tool for solving problems in equational theories that can be presented by conditional term rewriting systems. The user interface allows the user to specify the configuration of the distributed system, to type in formulas in mathematical notation, to design his own equational theory, and to examine the evolution of the constraint solving process.

The system is currently under development in the frame of the joint project, having as collaborators the institute RISC-Linz (Bruno Buchberger, Wolfgang Schreiner, Hoon Hong, Mircea Marin) and SCORE (Tetsuo Ida, Aart Middeldorp). It is available as a collection of *Mathematica* packages and notebooks that also represent the documented code of CFLP.

#23
Transformation of Logical Specification into IP-formulas

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Abstract
The classical algebraic modelling approach for integer programming (IP) is not suitable for some real world IP problems, since the algebraic formulations allow only for the description of mathematical relations, not logical relations. In this paper, we present a language L+ for IP, in which we write logical specification of an IP problem. L+ is a language based on the predicate logic, but is extended with meta predicates such as at least(m,S), where m is a non-negative integer, meaning that at least m predicates in the set S of formulas hold. The meta predicates are introduced to facilitate reasoning about a model of an IP problem rigorously and logically. Using *Mathematica*, we can represent the logical formulas, called L+ formulas efficiently and completely. But the L+ formulas can not directly executed by IP solvers. So, we need to translate L+ formulas into a set of mathematical formulas, called IP-formulas, which most of existing IP solvers accept. With *Mathematica*, we define a set of transformation rules and transform L+ formulas into IP-formulas and finally simplify the IP-formulas in *Mathematica*. We implement the transformation algorithm in *Mathematica* 3.0. Also, by using Mathlink and CGI programming, we develop a Web-based interface to support the system with modelling language, transformation of IP, and IP solvers. This provides a Web-based client-server model, in which the power of high level IP modelling and high performance IP solving can be integrated and developed for a wide range of business users to solve large scale decision making problems. The primary experiment indicates that *Mathematica* is powerful for representing logical formulas and for the transformation of formulas and that Mathlink is convenient for connecting *Mathematica* to other software platforms.

#24
Interactive Graphics Using MathLink

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Abstract
Interactive graphics effectively expresses the dynamic structure of a model, and helps improve the intuitive understanding of concepts in the sciences. Also it can be used as a user interface tool.

Presently, *Mathematica* graphics are not interactive but instead are static and merely a side-effect of computation. It is not possible to select a part of a graphic using the mouse, nor can one make interactive dynamic relationships between graphical elements.

The graphics system introduced here is a MathLink program for creating interactive graphical objects which the user can manipulate. In this program,

- all graphics — point, line, curve, and text — are represented as objects which can be manipulated by mouse;
- all graphics are manipulated by a kernel evaluation;
- the relationship between objects can be defined as *Mathematica* functions.

This paper outlines the system and discusses its design and implementation.

#25
Testing Chaos and Fractal Properties in Economic Time Series

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Abstract
Search for empirical evidence of chaos and testing fractal and other statistical properties in the framework of time series analysis are carried on as a preparatory step in order to apply these concepts to data

proper of Financial Markets and deal with the puzzling failure of traditional economic theories. Concepts like correlation dimension and Lyapunov exponents are discussed and simple *Mathematica* programs are given for their evaluation. Before their application to real economic data, a test on a well known nonlinear dynamical system, through the correspondent reconstructed phase space and time series, is carried out.

#26
The Visualisation of Different Fiber-optic Loop Interferometer Applications Work by Applying 3D *Mathematica* Graphics

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Abstract
The usefulness of the *Mathematica* 3D visualisation and animation for investigation of fiber-optic loop interferometer in different engineering applications work is presented. The Jones' matrix calculus has been used for description as fiber-optic elements as all interferometric systems. Such uniform description gives a possibility to obtain analytical relations, which depend on fundamental system parameters. Unfortunately, these relations are usually too complicated for physical interpretation. The possibility of their analyse by the animated 3D graphics is also shown in the present paper.

#27
Dynamic Optimization and Differential Games with Applications to Economics

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Abstract
In IMS'97 we showed several packages for: (1) solving simultaneous equations in real domain; (2) obtaining necessary conditions of constrained static optimization problems; (3) solving simultaneous nonlinear equations approximately; (4) determining signs of expressions; (5) solving differential equations approximately; (6) solving boundary-value problems approximately; and (7) solving dynamic optimization problems approximately. We also presented applications of these packages to urban economic models[1]. However, applications are limited in the package for solving dynamic optimization problems, since there exist many problems for which the package cannot be applied. In this paper, we revise and expand it so that it can be widely used. By further expanding the package above, we can solve differential games. We first show a solution of simple differential games with *Mathematica* and view the optimal trajectories. We then present packages for differential games by upgrading the dynamic optimization package, and show some applications to economic problems.

#28
The Application of Winding Numbers to the Interior Problem of Nonconvex Polygons and Polyhedra

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Abstract
The winding number for closed curves and polygons in the Euclidean plane is generalized to polyhedrain three-space. The winding number allows to decide wether a point is in the interior or exterior of a closed polygon/polyhedron. Polygons and polyhedra need not be convex and self-intersections are allowed.

#30
Sound as an additional learning tool

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Abstract
One of the advantages of using *Mathematica* in basic math courses is that it opens the possibility to create experiments in mathematics. In courses for students in electrical engineering at Chalmers University College we have used sound as an experimental tool to investigate aspects of mathematics that otherwise tend to elude students.

We give examples from two areas: ordinary differential equations and Fourier analysis. In the first example, we let *Mathematica* solve a differential equation with a parameter and then play the sound of the solution for different values of that parameter. The object is to find beat and resonance by listening. In the second example, we expand different functions in Fourier series and then listen to the functions and partial sums of these series.

#31
Mathematica-aided Education of Science-major Students

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Abstract
We will make a comprehensive report of our experience with *Mathematica*-aided education. Some students belonging either to Mathematics or Computer Science Department do graduation works using *Mathematica* as a language. We take up three subjects in those works.

#32
Mathematica as a Communication Enhancement Tool in the Classroom

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Kenji YOSHIDA, Konan Boy’s High School, JAPAN

Abstract
To use computers in Math classes, it is required that the computers can do Math well and are easy to use. Moreover, if they help communications between student and teacher and student and student, they enlarge the possibilities of interactions in classrooms.

However, technology of network software for education is not matured yet. Existing network softwares are sometimes powerless, and generally inflexible to modify and hard to collaborate with other educational softwares.

In this presentation, we demonstrate the network tools built on *Mathematica*.

Our goals are to provide tools;

1) help various type of communications such as; one-to-all, one-to-several,
and one-to-one,
2) help communications using not only text, but also various formats;
structured text, Mathematical formula, and pictures,
3) can be used easily by the student,
4) can be easily customized by the teacher for their special needs.

We report the effectiveness of the tools from the experiment and experiences.

#35
The Structuring Power of *Mathematica* in Mathematics and Mathematical Education

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Abstract
So far, *Mathematica* has been used mainly either in teaching as a support for classical courses, or in research as a computing environment. However, it could be assigned a wider and more profound role in mathematics and mathematical education. In particular, it could support, facilitate and accelerate the integration of algorithmic aspects into the corpus of mathematics.

#36
Mathematical Statistics with *Mathematica*

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Abstract
We present a unified approach for doing mathematical statistics with *Mathematica*. At one extreme, our package PDF empowers even the "statistically challenged" with the ability to perform complicated operations without realizing it. At the other extreme, it enables the professional statistician to tackle tricky multivariate distributions, generating functions, transformations, symbolic maximum likelihood estimation, unbiased estimation, checking (and correcting) of textbook formulae, and so on. By taking full advantage of the latest v4 Assumptions technology, the PDF package can produce exceptionally clean and neat symbolic output.

#37
Experiments in the Theory of Surfaces

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Abstract
The purpose of this article is to show the advantage of using *Mathematica* in the theory of surfaces. The examples in the classical differential geometry, namely the theory of curves and surfaces, have

been confined to a small group of calculable objects, because of the difficulty in evaluating geometrical quantities and solving differential equations explicitly. But *Mathematica* has made it possible to deal with wide range of objects and to perform experimental treatment of them, based on the power of numerical calculation, the NDSolve and NIntegrate commands in particular, and graphical visualization by the ParametricPlot command. I would like to introduce here several animations and figures that I use in my differential geometry class for undergraduate students. First of all, these graphics help students understand the basic notions of differential geometry. Secondly, we can experiment on geometry through these graphics. This article is one of the serial talks given by the author at Developers Conference 95, IMS 97 in Rovaniemi, and WMC 98 in Chicago. They are all targeted for the experimental usage of *Mathematica* on differential geometry. This time the topics are focused on the surfaces in the 3-dimensional Euclidean space. The commands are contained in six notebooks. If the commands in other notebooks cause any trouble, please restart *Mathematica*. See also the explanation of the notebooks.

#38
Interactive Image Processing FrontEnd using MathLink

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Abstract
It is not easy for commercial image processing softwares to add image processing effects or create new functions by combining tools. *MATHEMATICA* facilitates writing extensions because mathematical expressions of image processing are achieved and functions are defined simply. On the other hand, the existing Notebook FrontEnd lacks interactive processing such as direct manipulation of the graphics image with a mouse. Thus the authors have developed, using MathLink, an Interactive Image Processing FrontEnd (for short, Image Processing FrontEnd) with a variety of extensions. With this Image Processing FrontEnd, both the image processing can be done interactively and *MATHEMATICA*'s programming ability is effectively used. Advantages of this software are as follows:

1) A mouse and menus are available to draw pictures, similar to painting softwares.
2) It can read and write various graphic formats of image files.
3) Image processing operations are defined as *MATHEMATICA* functions.
4) Responses of a mouse and menus are defined as *MATHEMATICA* functions.

#41
On "*Mathematica* Natsu no Gakkou" (*Mathematica* Summer School)

Shinya Ohashi, Kashiwa Municipal High School
Kenji Yoshida, Konan Boy's High School

Abstract
Mathematica, a software system for mathematics, is well known among secondary school teachers in Japan. But not many of them have used *Mathematica* in the classroom or have developed teaching materials that use it. Even if some teachers have developed materials based on *Mathematica*, the use of those materials was limited to that teacher's school and seldom went beyond that school. There weren't any opportunites for most teachers to use such materials even if they existed. For teachers who are *Mathematica* novices, what is necessary is an opportunity to learn how to use *Mathematica* in the classroom and how to develop materials based on *Mathematica*.

A few years ago, we started *Mathematica* workshops for teachers of mathematics in several prefectures. Many teachers who attended it realized that it was possible to use *Mathematica* in future mathematics education. Unfortunately, these workshops were held in limited locations.

" *Mathematica* Natsu no Gakkou" is a project for providing the opportunity to develop *Mathematica*-related materials for teachers who do not have enough information on how to use *Mathematica*. The

first "*Mathematica* Natsu no Gakkou" was held in the summer of 1996 with the cooperation of Wolfram Research Asia Limited, the *Mathematica* User's Group in Japan, and *Mathematica* dealers in Japan. It was a three-day event. About 20 teachers of secondary education who were eager to develop *Mathematica*-related materials worked together. Each teacher completed one lesson based on an idea he/she brought. The third workshop was held last summer. So far, about 60 teachers have attended these workshops and various teaching materials have been developed. Anyone can download and use them freely. This, I think, is the first place where mathematics teachers from all parts of the country can communicate with one another.

The objective of this workshop is to develop materials which can be used in a classroom setting. Many types of materials have been made. They are:

1) materials to present ideas to students
2) materials to be used by students for self-teaching
3) materials to introduce *Mathematica* as a programming language

Thorough examination as to whether or not all of the materials developed at workshops can be used in all kinds of schools will be necessary.

In Japan, it seems to have become a requirement in the mathematics curriculum to not only utilize computers in mathematics, but to also teach programming in information science classes. Internet or LAN is becoming essential to schools. The possibility of introducing remote education or a credit system along with traditional classroom teaching is now being discussed. There will be a demand for teaching materials that students in any situation can use.

In this presentation, I want to talk about how the "*Mathematica* Natsu no Gakko" program is run. I also want to show some of the materials made there. Then I want to discuss the problems we confronted, and plans for developing teaching materials in the future.

#42
Teaching High School Mathematics over a Network with *Mathematica*

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Abstract
Many reports studying the use of *Mathematica* in the teaching of high school mathematics have been published. The majority of these reports focuses on the benefits of using *Mathematica* in the classroom. For example, the use of *Mathematica* enables students to perform complicated calculations, make assumptions and predictions based on the results derived from these computations, and then verify their findings. This represents a rather experimental method in mathematics instruction. Before the advent of *Mathematica*, students who were not proficient in calculations spent a majority of their time performing the computations, and very little time analyzing and processing the results. However, with the aid of *Mathematica*'s computational power, students can now focus on and enjoy the thinking and analytical part of the process– something which is completely independent of their ability to calculate. In this respect, *Mathematica* is a powerful tool for studying mathematics, especially useful for those students whose strength lies not in their computational capabilities. In this article, we will introduce the networked *Mathematica* system and how, in its development, we focused on its ease of use in classroom instruction.

#45
Pseudospectral Symbolic Computation for Financial Models

F.O. Bunnin, Y. Ren, Y. Guo and J. Darlington

Abstract
The modelling of financial markets as continuous stochastic processes provides the means to analyse the implications of models and to compute prices for a host of nancial instruments. We code as a symbolic computing program the analysis, initiated by Black, Scholes and Merton, of the formation of a partial di erential equation whose solution is the value of a derivative security, from the speci cation of an undelying security's process. The Pseudospectral method is a high order solution method for partial di erential equations that approximates the solution by global basis funcations. We apply symbolic transformations and approximating rewrite rules to extract essential information for the Pseudospectral Chebyshev solution. We write these programs in *Mathematica*. Our C++ template implementing general solver code is parameterised with this information to create instrument and model speci c pricing code. The Black-Scholes model and the Cox Ingersoll Ross term structure model are used as examples.

#46
Fractional Calculus: Application and Results

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Abstract
Over the last two decades anomalous relaxation and diffusion processes have been studied from both the experimental and theoretical point of view. Anomalous transport processes occur usually in so-called complex, i.e. in disordered systems. Here, we review some recent developments in modeling non-standard relaxation and diffusion equations based on Riemann-Liouville fractional calculus techniques. Closed analytical expressions of the solutions of such fractional differential equations are given in terms of Fox's H-functions. The analytical calculations carried out are fully supported by our *Mathematica* package *FractionalCalculus*.

#47
Compound Program Packages in Random Science Training and Technical Modelling

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Abstract
The present paper is presenting a review of our applications of computer algebra systems (CAS) [10, 11, 40] to different problems: for analysis of random phenomena in nonlinear dynamical systems and primary education in this sphere; for finding of control functions and for mathematical modelling of a mechanical objects movement. To archive final results in various tasks it is required to process a large number of mathematical calculations. Moreover the complexity of investigated objects requires to automate a procedure of model construction that results in the necessity of CAS using (together with numerical accounts) at all stages of scientific research.

#50
Recent Extensions in the Computation of Optimal Simplified Models for Systems with Time Delays

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Abstract
The earlier optimal method 1,2 for the simplification of single variable systems having ordinary polynomial denominators has been extended to the multivariable case encompassing situations where the original or simplified model may have delays in their states. The performance index used for optimization is the integral of the time-weighted squared error between the responses of the original and simplified models. The performance index is first expressed in terms of the simplified model unknown parameters and a minimization of this index gives the simplified model optimal parameters. The results of this work find additional application in obtaining the closed-form expressions of the error squared integral between the reference input and output response for certain closed-loop systems involving plants with time delays.

Key words: Model simplification, Multivariable time delayed systems.

#51
Mathematica-based Package for Studying Ordinary Differential Equations and for Analyzing the Learning Process

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Abstract
The design of a *Mathematica*-based learning environment for ordinary differential equations (ODE) is discussed. The ODE package consists of hypermedia based information about differential equations and a collection of different kinds of exercises. The purpose is also to collect data about the student's work and behaviour during the computer sessions using a recording system created as a part of the package. The data is used for analysing the learning process. The final goal is to get more knowledge about appropriate ways to use computer algebra systems like *Mathematica* in mathematics studies.

#52
An Example of Computer-based Study Material in Mathematics: Planning Principles and Realization

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Abstract
The development of software and network technology has given enormous possibilities to teaching and learning. At least, it is widely believed that there is a great potential. In practice, it is not so clear how these possibilities should be used. An experiment in the field of mathematics at university level is reported: philosophy, planning principles, realisation, future. The core of the experiment is a study package based on hypertext, web browsers and computer algebra systems.

#53
Computing of Curvature Invariants in Arbitrary Dimension

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Abstract
The MathTensor software enables to calculate the curvature invariants of differentiable manifolds in the case when the dimension of manifold under consideration is a concrete number. Proving the existence of geometric objects it is important to have examples for all dimensions n; greater or equal to some n0; i.e. when the dimension is just a symbol. The aim of the paper is to show how, using MathTensor and *Mathematica* commands, one can define the covariant and contravariant components of the metric tensor. Then we will show how to define different curvature tensors and how to execute the calculation of their components.

#54
Normal Lines Drawn to Ellipses and Elliptic Integrals

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Abstract

Consider the ellipse in the standard position, and P, an arbitrary point on the plane. Finding the minimum distance from P to the ellipse is a well-known problem. One can easily show that the minimum distance path lies along a normal line to the ellipse, passing through P. This paper deals with a study of all the normal lines drawn from point P to the ellipse. We have used the computer algebra system *Mathematica* to illustrate and discover several aspects of these normal lines. *Mathematica* can be used in contemporary mathematical research and education in more than one way: as a computational, visualization, experimentation and a conjecture forming tool. The paper well illustrates such usage via a study of the normal lines to the ellipse. We used *Mathematica* version 3.0 on a Windows 95 platform.

#55
Musical Representations of the Fibonacci String and Proteins Using *Mathematica*

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Abstract
Mathematica has been used in a novel application to produce music from algorithmic strings derived from the Fibonacci string and from amino acid sequences found in the primary structure of proteins. The musical output provides an example of a new dimension in assessing organizational patterns in complex *Mathematica*l strings and in chemistry for macromolecules. Structures and data can be heard as an adjunct to being visualized.

#56
System Stochasticity: Discrete Formulation with *Mathematica*

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Fortran/C driven completely numerical schemes have been extremely successful for deterministic systems that are discretised by the finite and boundary element methods. Randomness in material constitutive properties and boundary geometry introduces second order effects in responses. Real world geotechnical and bioengineering models, which exhibit severe randomness, initiated this research.

Displacement based finite elements and boundary elements with Green's functions for homogeneous media are inadequate for system stochasticity problems. Algebraic formulations become essential to account for the aforementioned second order effects. The author employed the *Mathematica* programming language to carry out the analytical steps. The subsequent recording of the symbolic routines into C++ were undertaken since the Monte Carlo simulation technique remains to be the only viable choice when large spatial variability is encountered in practical situations.

Research in stochastic finite/boundary element methods led to the following concepts in engineering mechanics: defect-free quadrilateral elements, stochastic Green/shape functions and Almansi boundary value problems.

Additional information can be found in various papers mentioned in author's home page: http://www.columbia.edu/~gd18.