The visualisation of different fiber-optic loop interferometer applications work by applying 3D Mathematica graphics

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Abstract

The usefulness of the Mathematica 3D visualisation and animation for investigation of fiber-optic loop interferometer in different engineering applications work is presented. The Jones’ matrix calculus has been used for description as fiber-optic elements as all interferometric systems. Such uniform description gives a possibility to obtain analytical relations, which depend on fundamental system parameters. Unfortunately, these relations are usually too complicated for physical interpretation. The possibility of their analyse by the animated 3D graphics is also shown in the present paper.

1 Introduction

The fiber-optic loop interferometer is used as a sensor of different physical fields. Depending on its configuration it can be applied as fiber-optic gyroscope [1], distributed sensor used in security system [2], vibrometer [3], ellipsometer [4], and the other devices [5]. The correct design and manufacturing of above devices requires precise knowledge concerning physical foundations of an action of given fiber-optic interferometer. Despite the fact, that those systems are only fiber-optic analogues of perfectly known classic interferometric systems, properties of optical fiber introduce significant modifications of a system work. Optical fibers and fiber-optic elements in general exhibit variable birefringence. In case of an application for a construction of interferometric system, it introduces the effect of polarisation on interference phenomena. In contrary, optical paths of classic interferometers are usually isotropic or exhibit constant birefringence. This is the main difference between bulk- and fiber-optic interferometers. For those reasons, coherence behaviour for the first and polarisation behaviour for the second kind of interferometer has the main influence on the system action [6].

In all above mentioned applications, the different fiber-optic elements are connected in various order, and for this reason different parameters affect system action. The investigation of such system is difficult, and then special numerical procedures must be solved. From physical point of view, the Jones’ matrix calculus [6] giving an information about polarisation behaviour of optical systems work, is one of the most useful for such investigation. In this formalism, the interferometer transfer function for the fiber-optic interferometer, which may be described as [7]:

\[ I = 0.5 \left[ \pm V \cos(\phi + \phi_o) \right] \quad V = \text{abs}(m), \quad \phi_o = \text{arg}(m), \quad \text{where} \quad m = E_{in}^{+} M_r^{+} M_s E_{in}, \] (1)
is dependent on polarisation behaviour of input beam (described by the Jones’ vector $\mathbf{E}_m$) and reference and signal arms of the interferometer (described by the Jones’ matrix $\mathbf{M}_r$ and $\mathbf{M}_s$), respectively. In the above notation, $V$ describes scale factor (the system sensitivity) while phase coefficient is a sum of phase $\varphi$ generated by measurable factor and additional shift $\phi_o$ (the system bias or drift). Moreover, the sign $^+$ stands for Hermite conjugation.

The above description is true for all fiber-optic interferometer configurations such as: Mach-Zehnder, Michelson, Fabry-Periot, ring resonator, differential and loop (usually named Sagnac – FOSI). The usefulness of the Mathematica for studying similarities and differences of all above configurations has been shown at last IMS [8]. The investigation only one of them – the FOSI, but for different technical applications with the aid of the Mathematica is described in this paper.

2. The basic behaviour of the FOSI

The main difference between FOSI and other kinds of interferometer configurations consists in the use of only one piece of an optical fiber for reference and signal beams. From this reason, the matrices describing those beams are usually (but not always) in transpose relation:

$$\mathbf{M}_r = \mathbf{M}_s^T$$

Such construction has the main influence on system behaviour – practical good operation in all types of sources and small sensitivity on environmental disturbances. Unfortunately, the general sensitivity of this interferometer configuration on polarisation property (Eq.1), is serious problem that must be solved [8]. The application of the polarisation controller (PC) in interferometer loop is one of usually used methods for compensation of polarisation changes in optical fiber. As one can see from simulation (Figure 1), the proper system adjust by PC gives a possibility for optimisation (maximum value of transfer function) of output signal obtained from system.

![TRANSFER FUNCTION OF FOSI](image)

Figure 1: The correction of polarisation properties of FOSI by means of PC. Parameters $\alpha$, $\beta$ are the angles position of PC loops [8].

However, if such FOSI detects the external perturbation (different value of $\varphi$ in Eq.1), the polarisation system properties affect the measurement. This fact is clearly seen from the
animation of last showed 3D graphical representation of the transfer function. As one can see from this animation (Figure 2), the different value of detected phase $\varphi$ give not only changes output intensity but also changes surface shape ($\alpha$, $\beta$). That situation means that it is impossible unambiguously determine the phase $\varphi$ (measuring disturbance) only by a registration of output intensity from system. From this reason, the special method of polarisation control must be used [7].

Figure 2: The part of the animation of FOSI response for different values of the detected phase $\varphi$.

The mutual relation between a light source and polarisation property of optical fiber applied for the FOSI, can also be an additional disturbance source of system work. Assuming that, the reference wave $E_r$ remains undisturbed, and all disturbances (connected with source characteristic $g(\omega)$ and FOSI construction) are introduced to signal wave $E_s$ we can describe these waves as Jones’ vectors in the following form [7]:

$$E_r = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{i(\omega r - \beta r)}; \quad E_\varphi = R(\eta) \begin{bmatrix} \cos \varphi e^{i(\Delta + \varphi)} \\ \sin \varphi e^{i(\Delta + \varphi)} \end{bmatrix} e^{i[\varphi(\omega - \beta(z + L)) + \varphi + \zeta]}$$

(3)

where: $\omega$ is optical frequency, $\beta$ - the wave propagation constant in the optical fiber, $(\varphi, \Delta)$ - the
rotation angle of polarisation plane and a change of phase retardation between field components, rotation angle of polarisation plane and the phase retardation between field components in reference beam, respectively. It is also assumed that reference wave is linear polarised, and the all sensor loop length \( L \) is included in signal beam, which included measurable phase shift \( \varphi \). Moreover, the parameters \( (\eta, \varepsilon, \xi) \) are connected with perturbation of output state of polarisation and \( (\phi, \Delta) \) and measurable phase \( \varphi \) that depend on source characteristic \( g(\omega) \), which for multimode semiconductor laser with spectrum distribution modelled in Gaussian shape (Figure 3) is [9]:

\[
g(\omega) = \sum_{i=-\infty}^{\infty} a_i g_i, \quad g_i(\omega) = \frac{\ln 2}{\sqrt{\pi} \delta \omega_i} \exp\left[ -\frac{1}{4} \omega - \omega_i \right] \sum_i a_i \delta \omega_i d\omega = 1 \quad (4)
\]

where: \( 2n+1 \) – numbers of single peaks each of them on the central frequency \( \omega_i \) and of the full width at half maximum (FWHM) \( \delta \omega \).

Figure 3: The model of Gaussian shape spectrum characteristic of multimode semiconductor source.

Above description of interfering beams gives in the end the following relation for transfer function [9]:

\[
I = |(E_{r} + E_{s})|^2 = 0.5 \left[ 1 + T(\phi)T(\phi + 2\pi n) \exp\left\{ \cos \phi + T(\Delta + 2\pi n) \cos(\varphi + \Delta) \cos \phi + \right\} \right] 
\]

where: \( T(\chi) = \exp[ - (\delta \omega^2 \chi^2 / 4 \ln^2 \omega_0^2) ] \), and \( \delta \omega \) is FWHM of single spectrum peak on the central frequency \( \omega_0 \).

The exponential relation depended on the rotation angle of polarisation plane \( \phi \) and the phase retardation between field components \( \Delta \) can be significant. In general (for \( m=0 \)), the influence of polarisation parameters of interacting beams is similar to that one shown in [8] for
monochromatic source (high coherence laser). If these beams have an orthonormal states of polarisation (SOPs) (i.e., $\Delta=0$, $\phi=\pi/2$), the detected signal disappears, whereas for the some SOPs output signal has the maximum value (see first picture in Figure 4). The essence of the problem is connected with global value of argument of T-function. This function is a result of random perturbations of SOP interacting beams caused by low coherence source. The $m$ parameter in this functions is loop length $L$ to beat length $L_b$ ratio, where last described polarisation property of optical fiber ($L_b=0.05 – 0.75$ m is chosen for a standard single-mode fiber). Hence, for FOSI with long loop length the $m$ parameter can achieve such large values so T-function goes to zero. In such situation (Figure 4) the output signal from FOSI disappearing regardless on interacting beams SOP. The described physical situation, named polarisation fading, can be beautifully presented by applying the animate property of the Mathematica (Figure 4).

Figure 4: The part visualisation of polarisation fading phenomena for FOSI with low coherence semiconductor source ($\omega_0=1500$ THz, $\delta \omega=21$ GHz and $2n+1=17$ modes).

The above problem can be dramatically important for FOSI application using long length of loop, such as distributed sensor or gyroscope, and may be solved by applying highly-birefringent (Hi-Bi) optical fiber or depolarised light.
3. The usefulness of Mathematica for polarisation research of different FOSI

The 3.01 version of the Mathematica has been used for study of polarisation properties of different FOSI applications developed in Physics Applications Department of IAP. The main of them were FOG – fiber-optic gyroscope, FOSESO – fiber-optic system for external security of object, FOPAMS – fiber-optic phase-amplitude measurement system, FOSE – fiber-optic Sagnac ellipsometer. In all of them, the equivalent lumped element representation ELER has been used for description of elementary fiber components. Such description presents the element as finite product of four different type matrices. These matrices named: rotator \( R \), retarder \( G \), polarizer \( P \), and absorber \( A \), depend on main constructional parameters of describing fiber-optic elements. In spite of simplicity of mathematical description of these matrices (2x2 range), their large numbers is the main difficulty with obtaining the analytical final relation. The used standard Mathematica matrices symbolic calculus gives an easy way for obtained final relations. Moreover, the used standard Mathematica software packages, especially 3D visualisation and animation, give easy way for tracking influence of changing different construction parameters of various elements on all system work in given sensor configuration.

In the next paragraphs, the certain results of Mathematica application for FOG and FOSE investigation are presented as the examples.

3.1 The fiber-optic gyroscope

The fiber-optic gyroscope FOG (Figure 5), developed in IAP, contains the optical head constructed according to so-called minimal configuration. It is (Figure 6) a collection of sixth optical elements (PO - polarizer, C1, C2 - two couplers (CR – reflected and CT – transmitted way), PM – phase modulator, SL – sensor loop, and PC – polarisation controller) and fifth optical connectors – R1, ..., R5. Additionally, the optical head contains a source S and a detector D.

The application of the Jones’ calculus for this system gives the following description of two interfering beams:

\[
M_s = CR_1^T.R_2^T.PO^T.R_3^T.CR_2^T.R_5^T.INV.PM.SL.PC.R_4.CT_2.R_3.PO.R_2.CT_1.R_1
\]

\[
M_r = e^{i\pi/2}CR_1^T.R_2^T.PO^T.R_3^T.CT_2^T.R_4^T.PC^T.SL^T.PM^T.INV.R_5.CR_2.R_3.PO.R_2.CT_1.R_1
\]
The description for each of these seventeen matrices is given as finite combination of basic matrices depending on constructional parameters:

Clear[{G, R, P, A, Dc, Inv, Ein}];

\[ G(\phi) := \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}; \] (* Phase retarder *)

\[ \mathbf{R}(\theta) := \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}; \] (* Rotator *)

\[ \mathbf{F}(\mathbf{\kappa}, \epsilon) := \begin{pmatrix} \mathbf{k} \cdot \epsilon \\ 0 \end{pmatrix}; \] (* Polarizer *)

\[ \mathbf{A}(\mathbf{\rho}) := \begin{pmatrix} \mathbf{0} \\ \rho \end{pmatrix}; \] (* Absorber *)

\[ \mathbf{D}(\phi) := e^{-i\phi} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \] (* Constant phase delay *)

\[ \mathbf{I} := \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \] (* Inversion matrix *)

\[ \mathbf{E}(\phi, \Delta) := \begin{pmatrix} \cos(\phi) \\ \sin(\phi) e^{i\Delta} \end{pmatrix}; \] (* Input Polarisation <two parameters: \phi - rotation angle, \Delta - retardation> *)

Clear[{SL, RM, FC, FO, CT, GR}];

(* Sensor Loop <six parameters: L - loop length, B - fiber birefringence, h - crosstalk, \lambda_L - depolarisation length, \phi_1, \phi_2 - ends phase disturbances> *)

\[ \mathbf{S}(\mathbf{L}, \mathbf{B}, \mathbf{h}, \mathbf{\lambda_L}, \phi_1[\phi_2]) := \mathbf{G}(\mathbf{B}, \mathbf{L}, \mathbf{h}) \cdot \mathbf{R}(\mathbf{\phi_1}, \mathbf{\phi_2}) \]; (* Phase Modulator <three parameters: N - loop numbers, \mathbf{R_m} - pet radius, \mathbf{B} - fiber birefringence> *)

\[ \mathbf{P}(\mathbf{N}, \mathbf{R_m}, \mathbf{B}) := \mathbf{G}(\mathbf{N}, \mathbf{R_m}, \mathbf{B}); \] (* Polarisation Controller <two parameters: \alpha - first loop angle, \beta - second loop angle> *)

\[ \mathbf{F}(\alpha, \beta) := \mathbf{R}(\mathbf{0.92}, \beta - \mathbf{\alpha}, -\mathbf{\alpha}); \] (* Polariser <one parameter \epsilon - extinction level> *)

\[ \mathbf{E}(\epsilon) := \mathbf{F}(1, \epsilon); \] (* Coupler (two CL and C2): Transmission way - CT, Reflection way - CR <two parameters: L\mathbf{c} - coupler length, B\mathbf{c} - coupler birefringence, K - interfiber coupling coefficient> *)

\[ \mathbf{C}(\mathbf{L}, \mathbf{B}, \mathbf{K}) := \mathbf{A}(\mathbf{G}(\mathbf{K}), \mathbf{L}, \mathbf{G}(\mathbf{B}, \mathbf{L})/2); \] (* Connectors RL - RS in form of Matrices R[\mathbf{R}] <five elements: \mathbf{R_1}, ..., \mathbf{R_5} - elements axes allignment> *)

Substituting the above ELER to Eqns 6 and 7 allows through Eq.1, for numerical simulation of the FOG operation. The transfer function achieved in this case depends on a group of 25 parameters characterising fiber elements. The additional 11 parameters secure generality for simulation (i.e. a possibility of sensitivity and loss of system calculation, etc). Then by suitable changes of 36 different parameters above simulation give possibility for an investigation of all FOG.

Figure 7 presents the results of influence the misalignments of birefringence axes in connections of the coupler with polarizer (matrix \textbf{R2}) and coupler with sensor loop end (matrix \textbf{R5}) on system operation. Physically it seems that the SOPs of the beam input at the polarizer and from one end of the coupler to the sensor loop are mismatched.
3.2 The fiber-optic Sagnac ellipsometer

The possibility to detect the changes of full SOP in the system containing standard single-mode fiber, by appropriate applied modulation technique, is the FOSE system (Figure 8), recently investigated system in IAP. This system, schematically shown in Figure 9, has double helix coiled loop, two polarizer controllers PC2, PC3, and the classical fiber-optic phase modulator M - all in the loop, and the additional coupler C. The proper system work requires orthonormal SOP of two interfering beams. This is achieved by nulling the first modulation harmonics of the signal $I_1(t)$ from the detector D1 by rotating the controllers PC1, PC2 and PC3. From the physical point of view, in this state birefringence of the all-interferometer loop is equivalent to this of the $\pi/4$ angle rotator. Then the final system adjustment is achieved by the maximal increase of the second harmonics of the signal $I(t)$ on the detector D by the controller PC4.

For such a system, the Jones’ vector of the output beam $E$ on the detector D has the following form [4]:

$$E = \frac{1}{8} P \cdot R(\pi/4)R(\pi/4)D(\Phi \sin \omega t)\text{Inv} + \text{Inv}D[\Phi \sin \omega (t - \tau)]R(-\pi/4)E_{\text{in}}$$

(8)

where $D$ is the Jones’ matrix of the phase modulator with time dependent modulation of $\Phi \sin \omega t$, and $\tau$ is a retardation time in sensor loop. An $E_{\text{in}}$ is the Jones vector of the input beam with searched parameters of SOP - $\phi$, and - $\Delta$. 

Figure 7: Changes of intensity scale factor $V=\text{Abs}(m)$ and phase bias-offset $\phi_0=\text{Arg}(m)$ versus angle of the misalignment of element birefringence axis (matrices $R_2$ and $R_5$).
It is useful to search the analytical relation of output signal generated by detector $D$ using standard Mathematica symbolic calculation:

\[
\begin{align*}
R[a] &= \begin{pmatrix} \cos[a] & -\sin[a] \\ \sin[a] & \cos[a] \end{pmatrix}; \quad \text{(Rotator)} \\
ML &= e^{i\delta_0 \sin(\omega t)} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \text{(Phase modulator with modulation } \delta_0 \sin(\omega t) \text{ for } \omega \text{ beam)} \\
MR &= e^{i\delta_0 \sin(\omega (t-\tau))} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \text{(Phase modulator with modulation } \delta_0 \sin(\omega t) \text{ for } \omega \text{ beam)} \\
Ein &= \begin{pmatrix} \cos[\phi] \\ \sin[\phi] \end{pmatrix} E^x; \quad \text{(Input Jones vector)} \\
Et &= ML \cdot R[\pi/4] \cdot Ein; \quad \text{(Jones vector for output wave from loop in transmitted mode)} \\
Er &= E^{x*} \cdot MR \cdot R[-\pi/4] \cdot Ein; \quad \text{(Jones vector for output wave from loop in refracted mode)} \\
w &= Et + Er; \quad \text{(Detected amplitude (sum of above Jones components))} \\
w1 &= \text{Simplify}[w]; \\
P &= R[\pi/4] \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \cdot R[-\pi/4]; \quad \text{(Polarizer with angle } \pi/4 \text{)} \\
Eout &= P \cdot w1; \\
Ex &= \text{Simplify}[Eout[1]]; \\
Ex &= \text{Simplify}[Eout[2]]; \\
Ey &= \text{Simplify}[Eout[3]]; \\
i &= \text{Ex} \cdot \text{Conjugate}[Ex] + Ey \cdot \text{Conjugate}[Ey]
\end{align*}
\]

Finally, the calculation gives the electrical signal in the following form [4]:

\[
I(t) = E \cdot E^* \equiv 0.5 A \left\{ 1 - \sin(2\phi) \left[ \cos \Delta \cos \left( 2\Phi \cos \left( \frac{\tau \omega}{2} \right) \sin \left( \omega t - \frac{\tau \omega}{2} \right) \right] + \right. \\
\left. + \sin \Delta \sin \left( 2\Phi \cos \left( \frac{\tau \omega}{2} \right) \sin \left( \omega t - \frac{\tau \omega}{2} \right) \right) \right\}
\]  \quad (9)

where $A$ is the intensity-normalised factor.
It can be easily shown that choosing the working point on $J_0[2\Phi\cos(\tau\omega/2)]=0$, this signal contains only odd and even signal harmonics. Hence, the proper system performance requires appropriate amplitude of the phase modulation $\Phi$ generated by the modulator in the loop. Then using well-known technique for a detection of the first and the second signal harmonics, it is the possible to determine the searching parameters as:

$$
\Delta = \tan^{-1}\left[ \frac{A_{\omega}}{A_{2\omega}} \cdot \frac{J_2(\Phi)}{J_1(\Phi)} \right];
$$

$$
\phi = \frac{1}{2} \sin^{-1}\left[ \frac{1}{\lambda} \sqrt{A_{\omega}^2 + A_{2\omega}^2 - \sqrt{\cos^2 J_2(\Phi) + \sin^2 J_1(\Phi)}} \right]
$$

(10)

where: $A_{\omega}, A_{2\omega}$ are amplitudes of the first and the second harmonics of the signal.

From the formulae presented above it is easy to determine changes in SOP by estimating the values of $\phi$ and $\Delta$ in all ranges of their changes. Moreover, as one can see from simulations (Figure 10), the changes of $\Delta$ generally affect the signal harmonics ratio, whereas changes of $\phi$ affect the signal amplitude.

![Ellipsometer Signal Simulation](image)

**Figure 10**: Simulation of output signals for changing parameters of input SOP.

### 4. Concluding remarks

Sometimes, probably five – seven years ago, I was looked for new calculating programme. I need a programme, which has given as good matrix operation as good visualisation of obtained results. The standard numerical languages such as FORTRAN, C, PASCAL was not useful, because they have not standard visualisation procedures. Only the *MathCAD* and *Mathematica* gave possibilities satisfactory for me, and I bought the latter of them. I am fully pleased with this selection, form two reasons. For the first, *Mathematica* has built-in package that permits symbolic matrix operation on real and complex numbers. It is important possibility, because in Jones’ calculus results containing long product of trigonometric relations are often obtained.
simplification of such relations, half or all page long, by using human memory frequently gives some mistakes, whereas the artificial intelligence (Mathematica) is free from this problem.

For the second, the whole numerical calculus gives results, physical interpretation of which base on the examination of plotted relations. Usually it is possible to show influence only one parameter, as variable of a plot. The two variable parameters of the plot need 3D graphic. The point of view, direction of view and distance of viewer from plot have fundamental influence on observed results. The Mathematica offers this ability, which can be easy modified, also. Moreover, animate procedure included in Mathematica gives a possibility to use the next third parameter as a variable of simulation. The above aspects of Mathematica have been presented in this paper, mainly for investigation of polarisation properties of different FOSI systems work.

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References