Mathematica-based package for studying ordinary differential equations and for analyzing the learning process

Riikka Nurmiainen
Institute of Mathematics
Helsinki University of Technology
Finland
e-mail: Riikka.Nurmiainen@hut.fi

Abstract. The design of a *Mathematica*-based learning environment for ordinary differential equations (ODE) is discussed. The ODE package consists of hypermedia-based information about differential equations and a collection of different kinds of exercises. The purpose is also to collect data about the student's work and behavior during the computer sessions using a recording system created as a part of the package. The data is used for analyzing the learning process. The final goal is to get more knowledge about appropriate ways to use computer algebra systems like *Mathematica* in mathematics studies.

Introduction

Today, the availability of computer algebra systems such as *Mathematica* [8] has made it possible to bring realistic modeling into the classroom. Computers give teachers the possibility to spend less time on manipulative details and more time on mathematical concepts and applications.

Computers and software packages form an enormous potential for teaching and learning purposes. They have offered an opportunity to rethink the traditional ways on introducing the basic topics to students. Mathematical programs form a good means for demonstrations of mathematical phenomena. They are effective tools for making experiments and demonstrations in studying.

Unfortunately, the emphasis has mostly been on producing computer aided study

material and not so much has been done for developing material for different topics and for analyzing the learning process based on the material.

At the Helsinki University of Technology it has been gathered experience about using computers in mathematics studies [2, 5]. The study package introduced in this paper is a part of the larger research project called *MatTa* where the aim is not only to develop computer-based mathematical study material but also to study what kind of learning process should be supported [3].

Why differential equations?

The topics considered in our study package is ordinary differential equations (ODE).

Computer algebra system like *Mathematica* offers the possibilities to both analytic, numeric and qualitative techniques. It is not necessary to restrict to the differential equations with solutions in terms of usual elementary functions. It is also possible to focus less attention on manipulative skills and more attention on qualitative behavior of the solutions.

Mathematica gives possibility to computer-based explorations. Students can explore the properties of the solutions by

- graphing direction fields and phase portraits,
- using numerical methods,
- investigating the effects of changes in initial conditions and parameters,
- exploring the qualitative behavior of solutions and its relation to the behavior of a corresponding physical model.

If mathematical programs are used there is more time to concentrate on larger exercises and mathematical modeling. Even the simplest equations may correspond to physical models, for example, exponential decay and electronical circuits. Students can also build simple models of population growth, epidemics etc.

Learning process

In computer aided learning the computer is a medium to direct the student's learning process. As a base for planning principles of computerized study material

there should be a strong view on the learning process hoped-for. In the best case, computer can be used for supporting student's learning process.

The view on the learning process in the ODE package is based on the constructive learning theory. Constructivism emphasizes the important role of the learner in studying. Learning does not mean to remember a lot facts. It means to understand the things. For archieving this, the student has to construct his/her own knowledge from information. The learner acts as active constructor of knowledge.

The structure of the learning session is made of two separate part:

- First, the instructor introduces the subject of the day and gives the problems to be solved.
- Then, students work with the problems using the ODE package and *Mathematica*. The instructor observes the work of students. Students can ask instructor's guidance if necessary.

The aim is that the student searches information from the theory part on his/her own initiative, and starts to explore it and the problems with *Mathematica*.

ODE package

Ordinary differential equations are the topics of the mathematical study package because of the clear benefit from the computer in studying the subject.

Theory part

The theory part consists of hypermedia-based information about ordinary differential equations. The material has been written especially for the package. So, it is not a book transformed into electronical form.

The theory part is quite a compact presentation about ordinary differential equations. It consists of articles divided into modules. Modules and articles are linked together using the key words marked by the author.

Exercise part

The exercise part is a *Mathematica* notebook. It has quite a hierarchical structure. There is a list of exercises in the main notebook. Larger exercises are written in a separate notebook and they are linked to the main notebook.

The exercises are of different types. There are both open and guided exercises. The research level purpose is to find those exercise types supporting student's learning process in the best way.

The idea is that the student solves the exercises mainly using the basic commands of *Mathematica*. A short notebook-based *Mathematica* guide is available. There are also commands especially made for the package so that students can focus on the mathematical ideas instead of the studying more complicated commands and options. The aim is to learn mathematics not the technical details of the commands!

Test of the package

The test of the package is planned for October 1999 when some students in a mathematically oriented high school (Maunulan yhteiskoulu) have a course on ordinary differential equations; the ODE package will be the main study material. The aim is to collect data about students' work and behavior during the computer sessions using the recording system created as a part of the package. The data is used for analyzing the learning process of individual students.

The aim of the research is to find appropriate ways in using *Mathematica* in mathematics studies. The main task is to study if the students are capable to study in a computer aided learning environment of this type:

- What are the ways the students use *Mathematica* in their studies?
- What kind of technical difficulties may appear?
- Can students find information in hypermedia-based learning material?

Technical solutions of the package

Mathematica supports hypertext links and it is possible to typeset mathematical formulae into notebooks. *Mathematica* gives also possibility for writing study material with visualizations (pictures, animations) and it gives the student the possibility to experimentations. It is also quite easy to collect the data of the student's work in *Mathematica* session.

These are the main reasons for choosing *Mathematica* to the learning environment. The original plan has been to build the whole package top on *Mathematica*. It is also easier to the student if he/she can communicate with one system only.

Typesetting both ordinary text and mathematical formulae into the *Mathematica* notebook has not proved to be as flexible as expected. Therefore the source text of the theory part has been written in the mathematical typesetting system LaTeX [9]. Hypertext links are also defined in the LaTeX code. The purpose is to convert the contents of the LaTeX documents as automatically as possible to the *Mathematica* notebook form. Hopefully that will not mean copy–paste method and retypesetting formulae. If the conversion will not succeed some other possibilities must be considered.

One possibility is to convert the source text to pdf form and to use Acrobat Reader [11] for reading. That is the technical solution in the mathematical encyclopedia produced in the project *MatTa* [3].

Also some other possibilities have been thought. The material written in LaTeX can be converted to html form. But at this moment too many gif pictures are generated. In the near future the mathematical markup language MathML [10] will be the solution for this.

The use of a *Mathematica*-based commercial package VisualDSolve [6] is planned for visualizing differential equations in both theory and exercise part.

Recording mechanism

There are several ways of recording inputs and/or outputs of a *Mathematica* session in a file [5]. If *Mathematica* with the notebooks front end is used, all inputs and outputs can be saved in a notebook. When evaluating learning process it is necessary to record some other things too, for example time stamps. Because student can edit the notebook and correct syntax errors and wrong inputs it is impossible to reconstruct student's learning process by using only the notebook student has done. Student may also jump in a notebook so that the real order of the inputs can not be followed and analyzed. Of course, the notebook student has done gives valuable information about his/her work, and is an important part of evaluation the learning process.

Mathematica operates in a main loop: it waits for input, processes it, prints the result and then goes to waiting for input again [7]. *Mathematica* allows to insert functions to be applied to expressions at various stages in the main loop. The standard main loop begins by getting a text string of input. This text string can be processed by *Mathematica* by assigning a function as the value of the global variable `$PreRead`. `$PreRead` is done before the input is evaluated [1, 4, 7].

In the ODE package all inputs are recorded in a file using `$PreRead` command. *Mathematica*'s outputs are not recorded because of the huge length of the log file and it's always possible to rerun the inputs and see the outputs.

Every input is inserted into a log file with the value of `$Line` which gives the number of each input. The value of `SessionTime[ ]` gives the total number of seconds elapsed since the beginning of the current *Mathematica* session and `Date[ ]` gives the current local date and time.

There are some open questions to be solved:

- What kind of data ought to be collected? There are many useful functions which might be used in addition to those mentioned above. It might also be useful to record if there has been a syntax error in the input.
- How to follow the use of the hyperlinked theory part? This may be solved in the same way as above if the theory part can be converted to *Mathematica* notebook. In other case some difficulties may appear.
- How to save the cell where student is in the notebook? Has she/he jumped? Has the jumping confused student?

How to analyze the data recorded

The purpose of the recording mechanism is to evaluate how students interact with *Mathematica* and with the ODE package. For infering things about learning process some technics for analyzing recorded data must be developed. There are some indicators of learning process which can be analyzed from the data recorded:

- Which *Mathematica* functions did the student use? Did he/she use the commands expected? How many commands and how many times did he/she use?
- Were there common mistakes?
- How effective was the use of the tools? Did he/she use own variables for saving intermediate results?
- Were there any attempts to make own explorations?
- Have their skills to use *Mathematica* developed?

It is also possible to plot graph from the time stamps. Such information tells how much time elapsed between the presentation of a stimulus or a problem and the student's response. It gives some information about the activity of the student.

The logging data is not sufficient alone. It needs support of traditional methods like observation and interviewing of individual students, questionnaires and pre- and post-testing of students. One aim of the research is to study if it is possible to analyze learning process using only a logging mechanism.

References

[1] Kent P., Ramsden P., Wood J. and Noss R. Recording and Evaluating Student Interactions with Mathematica. Mathematica in Education and Research. Volume 5. 1996. http://metric.ma.ic.ac.uk/book/Logging.html

[2] Kivelä S. K. Producing computer-based material for mathematics studies. In Proceedings of the Second International Mathematica Symposium. Keränen V., Mitic P. and Hietamäki A. (editors). Computational Mechanics Publications. USA 1997.

[3] Kivelä S. K. Computer-based study material in mathematics: planning principles and realization. Proceedings of IMS'99. ims99paper52.pdf

[4] Maeder R. E. Programming in Mathematica. Third Edition. Addision-Wesley. USA 1997.

[5] Nurmiainen R. Computer exercises of basic courses in mathematics at the Helsinki University of Technology. In Proceedings of the Second International Mathematica Symposium. Keränen V., Mitic P. and Hietamäki A. (editors). Computational Mechanics Publications. USA 1997.

[6] Schwalbe D. and Wagon S. VisualDSolve. Visualizing Differential Equations with Mathematica. TELOS, The Electronic Library of Science. Springer-Verlag. USA 1997.

[7] Wolfram S. The MATHEMATICA Book. Third Edition. Mathematica Version 3. Wolfram Media. USA 1996.

[8] Mathematica: http://www.wri.com/

[9] LaTeX: http://www.tug.org/

[10] MathML: http://www.w3.org/Math/

[11] Acrobat Reader:
http://www.adobe.com/proindex/acrobat/readstep.html