# *CurvesGraphics*

*A free package for Advanced Calculus illustrations.*


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*Motivation*

As a teacher of Calculus and Mathematical Analysis at college and university level, I feel that *Mathematica*'s standard plotting routines ignore some of the more advanced needs in my field: curves with arrows along them, level sets on a surface, general parametric of contour curves on a surface. In the last several years I have set out to make a package of my own to address these problems, and any Mathematica user is welcome to use it freely.
I have chosen **CurvesGraphics** as the name of the package, because curves are its main focus, but an important and growing part is about surface manipulation. A more accurate descriptive name would be something like **CurvesArrowsSurfaceGraphics**; too long for my taste.

*General description*

The package **CurvesGraphics** adds the following capabilities to *Mathematica* graphics:

- *easily* placing **arrows along curves** in 2D or 3D space along the curves produced by Plot, ParametricPlot, ParametricPlot3D, and ContourPlot, and along the solutions to differential equations given by NDSolve;
- drawing **parametric or contour curves, or text, on a surface in 3D space, or the intersection curve of two surfaces**, all curves with optional arrows along them.

Number, positions, size, shape, style, color of the arrows and of the curves are customizable through options. The default values of the options have been chosen to give pleasing results in most typical cases that I could think of, with hardly any tweaking.

The package is self−sufficient in all its features but two: 3D arrows and 3D text, for which you will need to have David Park's **DrawGraphics** package installed in your *Mathematica* system. **DrawGraphics** users (like myself) will find that **CurvesGraphics** provides a **Draw** companion to all its Plot functions.

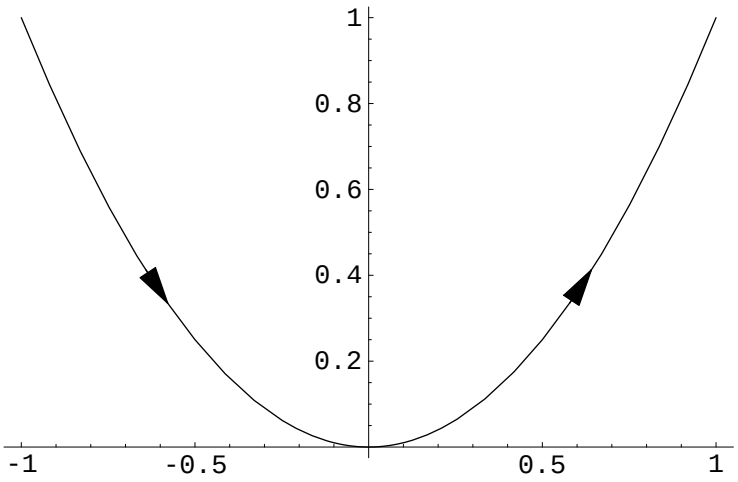
*Examples*

```
Needs["CurvesGraphics‘"];
```

## ■ Arrows on 2D curves

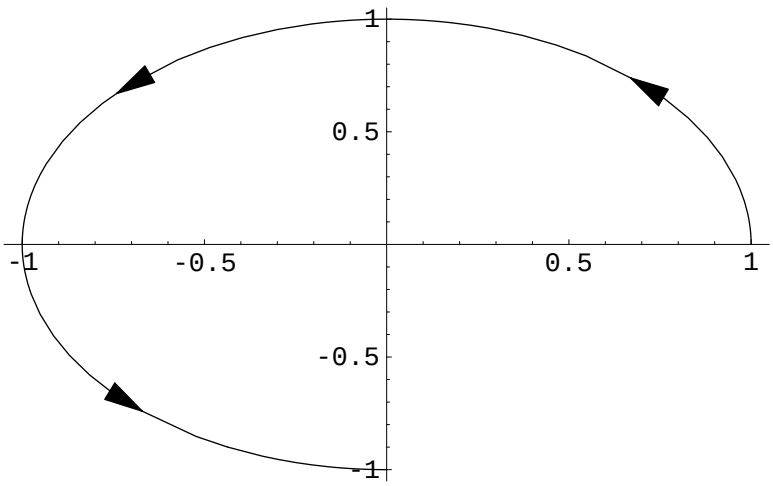
A parabola with arrow on it:

```
Plot[x^2, {x, -1, 1}, Oriented → True];
```



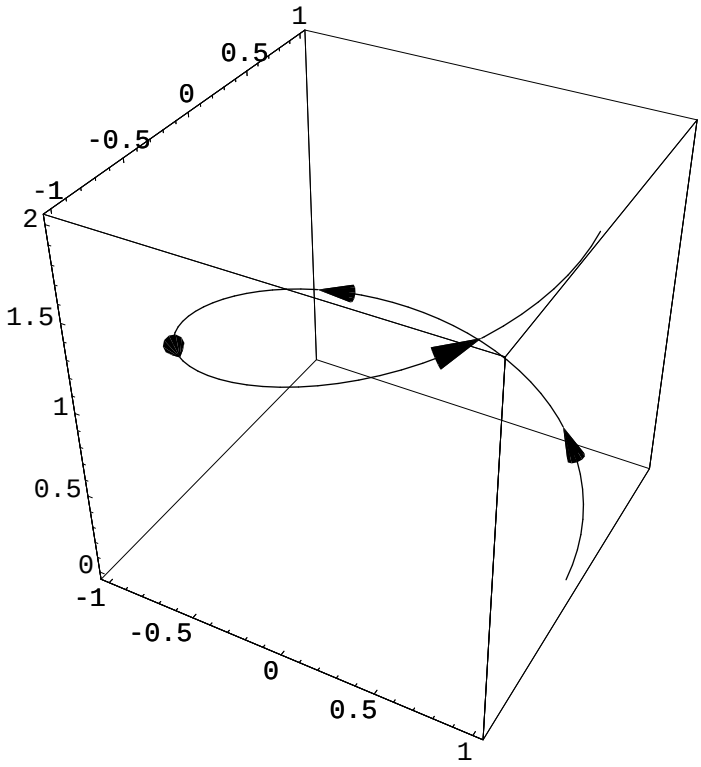

A circle with arrow on it:

```
ParametricPlot[{Cos[x], Sin[x]}, {x, 0, 3π/2}, Oriented → True];
```



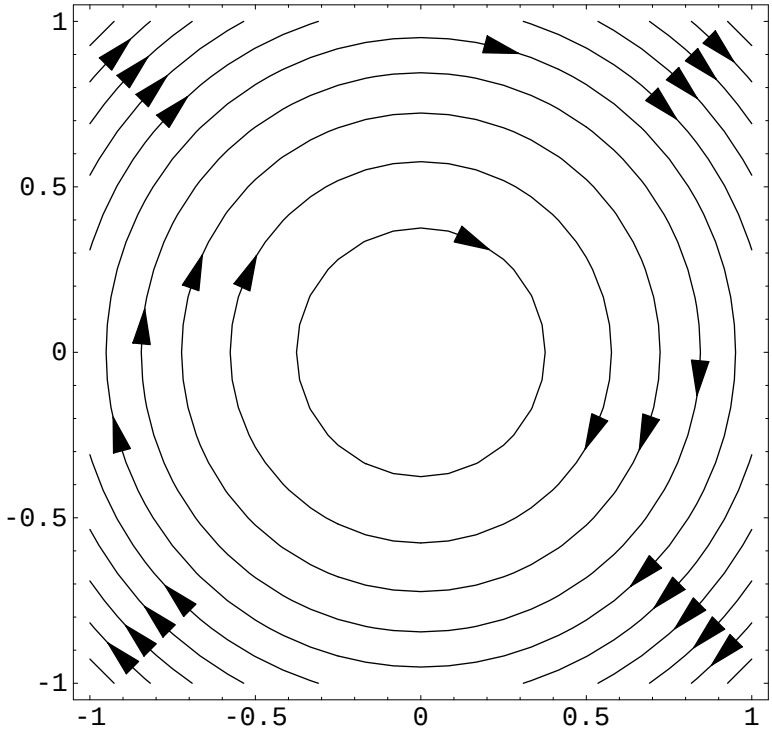

A helix with arrow on it:

```
ParametricPlot3D[{Cos[x], Sin[x], x/π},
 {x, 0, 2 π}, Oriented → True];
```




The contour lines of the function $(x, y) \mapsto x^2 + y^2$ :

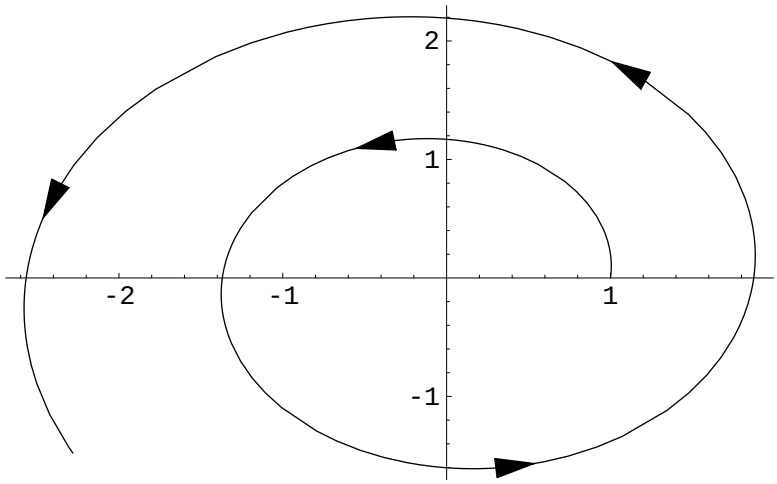
```
ContourPlot[x^2 + y^2, {x, -1, 1}, {y, -1, 1}, Oriented → True];
```




## ■ Plotting from within NDSolve

Numerical solution and plot of a differential equation in the plane:

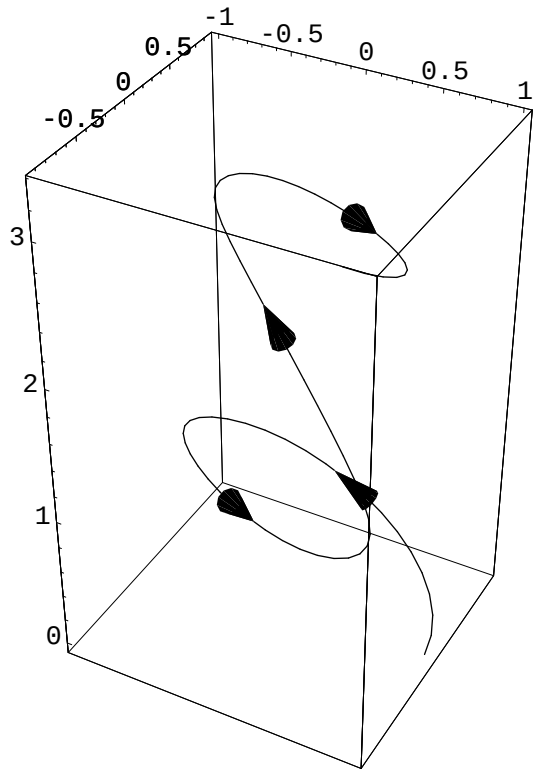
```
NDSolve[{x'[t] == x[t]/10 - y[t],
  y'[t] == y[t]/10 + x[t],
  x[0] == 1
  y[0] == 0},
 {x, y}, {t, 0, 10}, PlotSolution → True]
```




```
{{{x → InterpolatingFunction[{{0., 10.}}, <>],
   y → InterpolatingFunction[{{0., 10.}}, <>]}}, - Graphics -}
```

Numerical solution and plot of a differential equation in 3D space:

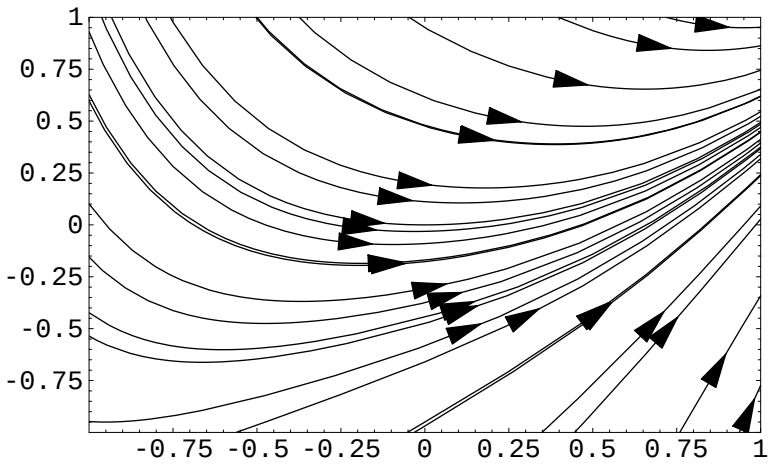
```
NDSolve[{x''[t] == -x[t] (1 + x[t]^2)
         y''[t] == -y[t] (1 + x[t]^2 - x[t]^4)
         z'[t] == 1/3
         x[0] == 1
         y[0] == 0
         x'[0] == 0
         y'[0] == 1
         z[0] == 0},
 {x, y, z}, {t, 0, 10}, PlotSolution → True];
```




## ■ Phase plots for differential equations

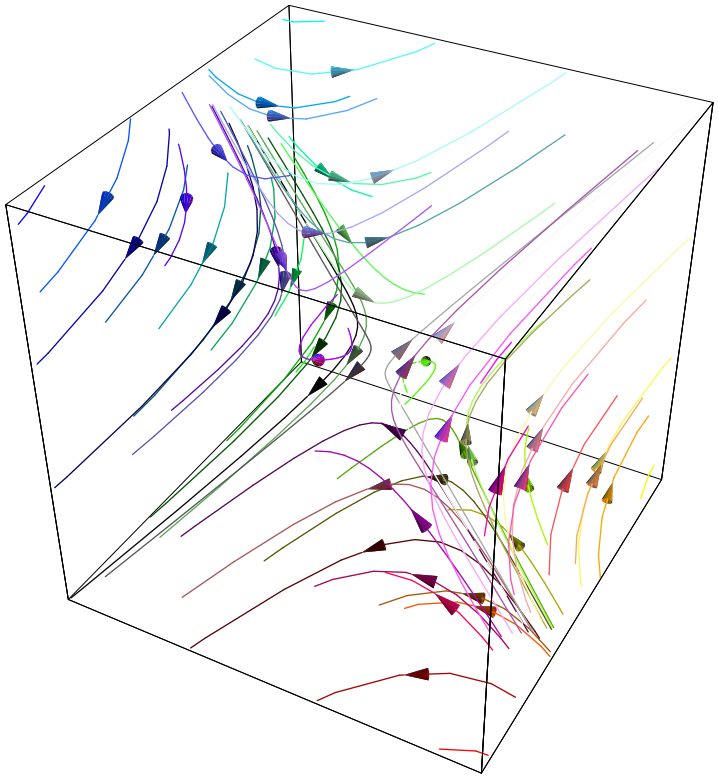
Force lines of a vector field in the plane:

```
PhasePlot[{1/(1+x^2), -y + x}, {x, -1, 1}, {y, -1, 1}, {-3, 3}];
```




Force lines of a vector field in 3D space:

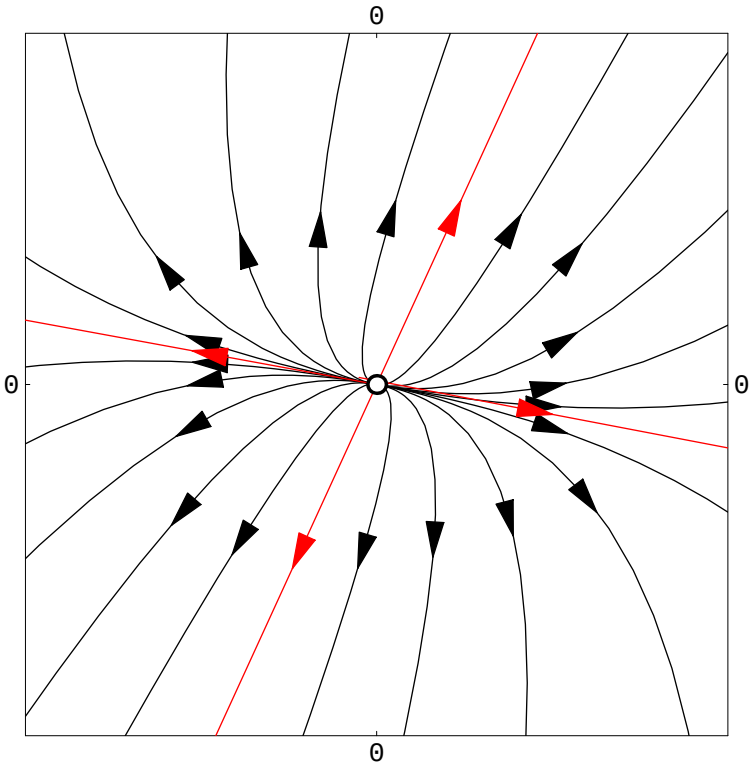
```
PhasePlot[{{1, 2, 3}, {4, 3, 2}, {3, 1, 2}}.{x, y, z}, {x, -1, 1}, {y, -1, 1},
 {z, -1, 1}, {-2, 2}, GridPoints → 4, PlotStyle → RGBColor];
```



Phase portrait of a linear autonomous differential equation in the plane:

```
LinearPhasePlot2D[{{1, 1/2}, {1/5, 2}}];
```

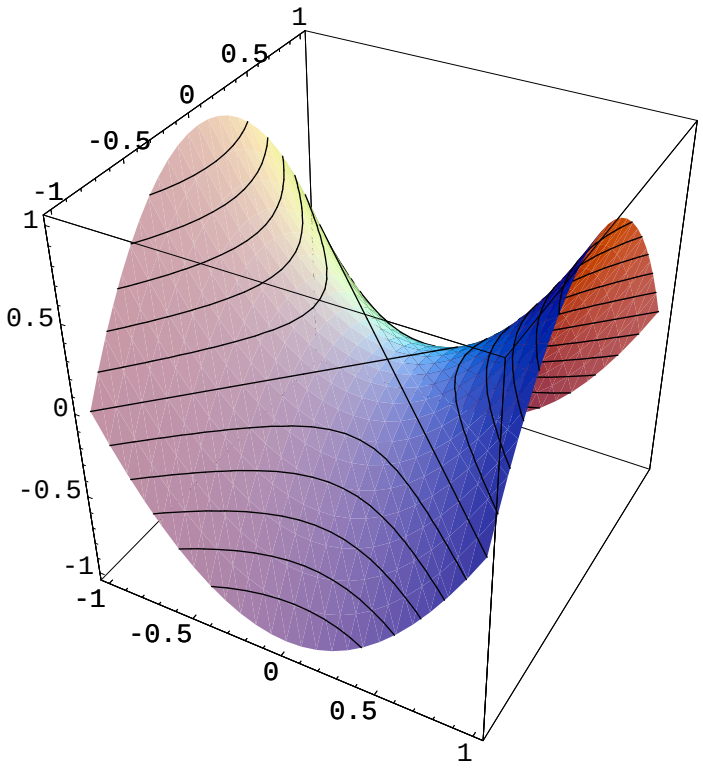
```
Repulsive node
```




## ■ Level lines on a surface

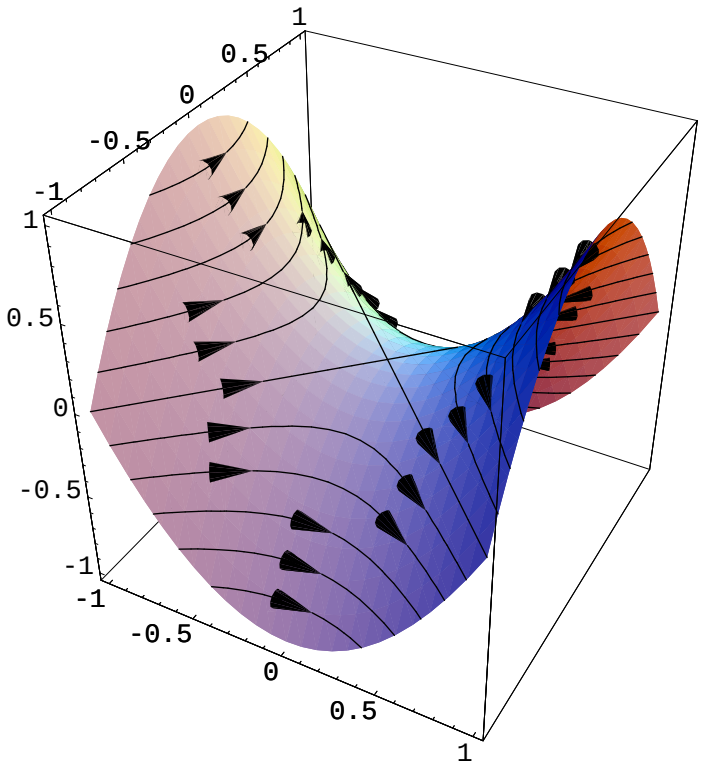
$z$–level lines on the surface plot of the function $(x, y) \mapsto x^2 - y^2$ :

```
LevelPlot3D[x^2 - y^2, {x, -1, 1}, {y, -1, 1}];
```




$z$–level lines, with arrows along them, on the surface plot of the function $(x, y) \mapsto x^2 - y^2$ :

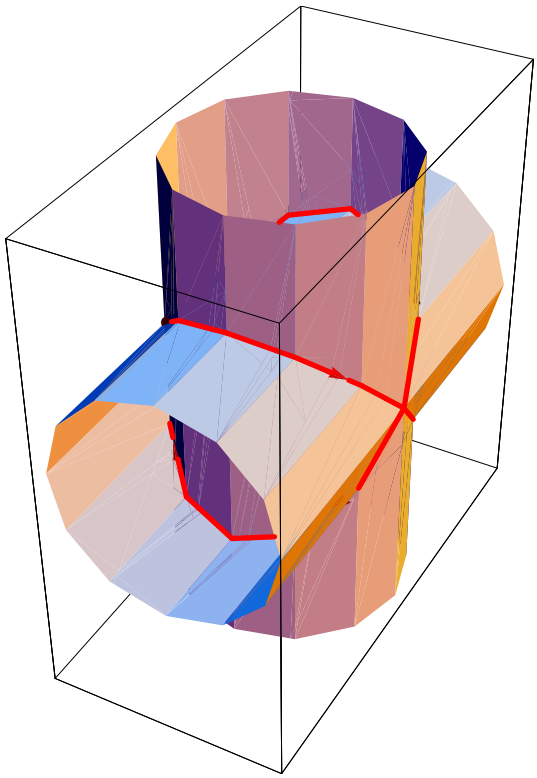
```
In[27]:= LevelPlot3D[x^2 - y^2, {x, -1, 1},
          {y, -1, 1}, Oriented → True, ArrowDensity → .8];
```




## ■ Intersection of two surfaces

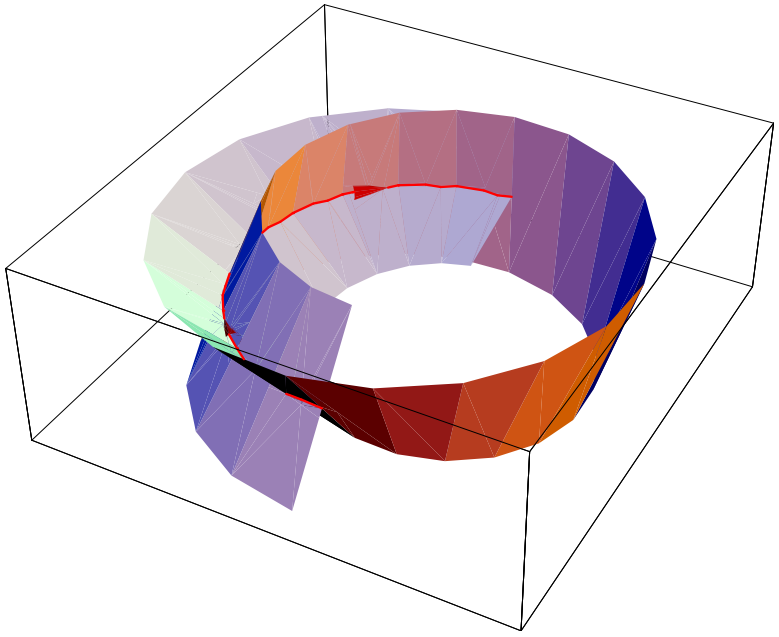
The intersection curve between two surfaces:

```
surf1 = ParametricPlot3D[{Cos[t], Sin[t], z}, {t, 0, 2 Pi},
   {z, -2, 2}, PlotPoints → {13, 2}, DisplayFunction → Identity];
surf2 = ParametricPlot3D[{Cos[t], y, Sin[t]}, {t, 0, 2 Pi},
   {y, -2, 2}, PlotPoints → {13, 2}, DisplayFunction → Identity];
SurfaceIntersectionPlot3D[surf1, surf2,
 PlotStyle → {Red, Thickness[.01]}, Oriented → True];
```



The self−intersection of a surface:

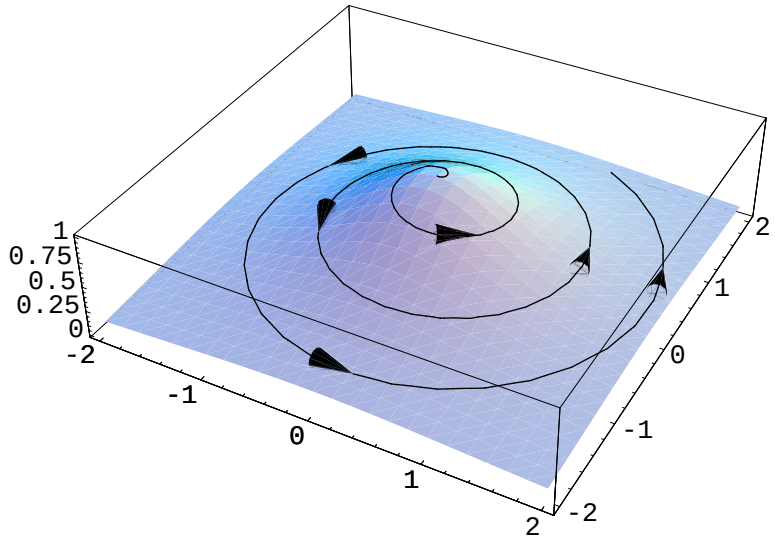
```
surf = ParametricPlot3D[
  {(1 + u Sin[ϕ/5]) Sin[ϕ], Cos[ϕ] (1 + u Sin[ϕ/5]), u Cos[ϕ/5]},
  {u, -1/2, 1/2}, {ϕ, -π, 2π},
  PlotPoints → {2, 31}, DisplayFunction → Identity];
SurfaceSelfIntersectionPlot3D[surf,
 PlotStyle → {Red, Thickness[.003]}, Oriented → True];
```



## ■ Parametric and countour lines on a surface

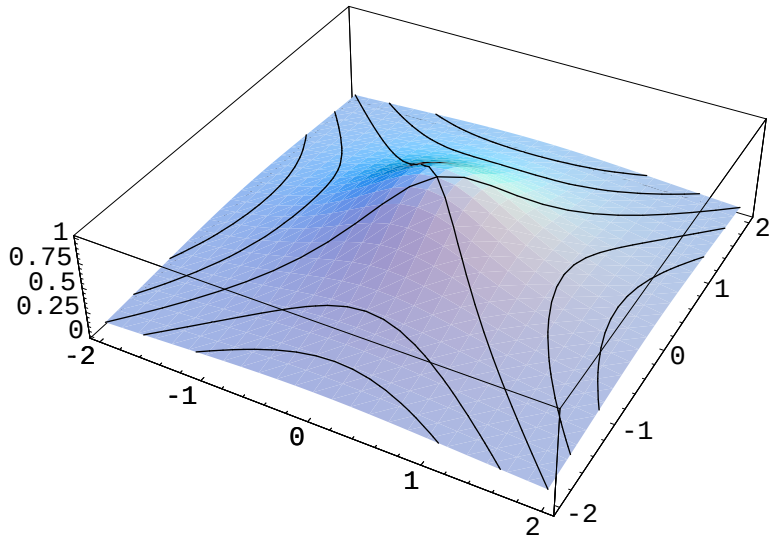
The parametric spiral $t \mapsto (t \cos 10\,t,\ t \sin 10\,t)$ "lifted" on the surface plot of the function $(x, y) \mapsto \frac{1}{1+x^2+y^2}$:

```
PlotCurveOnSurface3D[{1/(1 + x^2 + y^2), {x, -2, 2}, {y, -2, 2}},
 {{t * Cos[10 t], t * Sin[10 t]}, {t, 0, 2},
  PlotPoints → 100, Oriented → True}];
```



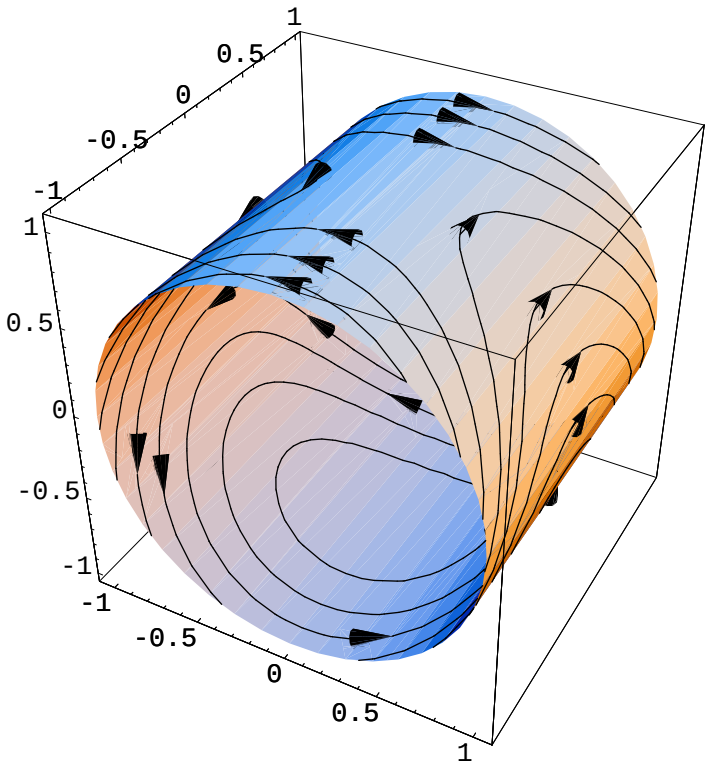

The contour lines of the function $(x, y) \mapsto x^2 - y^2$ "lifted" on the surface plot of the function $(x, y) \mapsto \frac{1}{1+x^2+y^2}$:

```
PlotCurveOnSurface3D[{1/(1 + x^2 + y^2), {x, -2, 2}, {y, -2, 2}},
 {x^2 - y^2, {x, -2, 2}, {y, -2, 2}, Contours → 5}];
```




The (oriented) contour lines of the function $(x, y, z) \mapsto z + y^2$ on the cylinder $x^2 + z^2 = 1$:

```
PlotCurveOnSurface3D[{{Cos[v], u, Sin[v]}, {u, -1, 1},
  {v, -Pi, Pi}, PlotPoints → {2, 41}}, {Sin[v] + u^2,
  {u, -1, 1}, {v, -Pi, Pi}, Oriented → True, PlotPoints → 35}];
```




A text written on a surface:

```
PlotCurveOnSurface3D[{-Sin[x] + Sin[y], {x, -4, 4}, {y, -4, 4}},
 Graphics[{Text["surface", {0, 0}]}]];
```