Robust Multi-Scale Optic Flow Using Toppoints

IMS ’08 Presentation
by Pieter van Dorst
20-06-2008
-Traditional Optic Flow
- Traditional Optic Flow

- Optic Flow from Toppoint Movement
- Traditional Optic Flow

- Optic Flow from Toppoint Movement

- Combining the Two Methods
- Traditional Optic Flow

- Optic Flow from Toppoint Movement

- Combining the Two Methods

- Results

motion?
-Traditional Optic Flow

-Optic Flow from Toppoint Movement

-Combining the Two Methods

-Results

-Future Work

A →

B →

A+B →

A+B → motion?

C → D → E → ...

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- Traditional Optic Flow
- Optic Flow from Toppoint Movement
- Combining the Two Methods
- Results
- Future Work
- Comparison With Other Methods

A →
B →
A+B → 
A+B motion?
C → D → E → ...
A+B > α,β?
**Traditional Optic Flow**

First frame of the movie:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
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<td>1</td>
<td>2</td>
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<td>2</td>
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<td>5</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Second frame of the movie:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>0</td>
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<td>1</td>
<td>2</td>
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<td>4</td>
</tr>
</tbody>
</table>
The Optic Flow Constraint Equation (OFCE) =

\[ u \cdot f_x + v \cdot f_y + f_t = 0 \]
Traditional Optic Flow

2 Problems:

<table>
<thead>
<tr>
<th>First frame of the movie:</th>
<th>Second frame of the movie:</th>
<th>No movement, changed brightness:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td>-1 -1 -1 -1 -1</td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td>-1 -1 -1 -1 -1</td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td>-1 -1 -1 -1 -1</td>
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<tr>
<td>1 2 3 4 5</td>
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</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td>-1 -1 -1 -1 -1</td>
</tr>
</tbody>
</table>

OR:

1: *The Brightness Constancy Assumption*
**Traditional Optic Flow**

2 Problems:

<table>
<thead>
<tr>
<th>First frame of the movie:</th>
<th>Second frame of the movie:</th>
<th>Another movement:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td>OR:</td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td>0 1 2 3 4</td>
<td></td>
</tr>
</tbody>
</table>

2: *The Aperture Problem*

Solution = Prior
Traditional Optic Flow

Formally:

Prior: Minimize image derivatives

Data term: OFCE: $u \cdot f_x + v \cdot f_y + f_t = 0$

Energy Functional: $E(u,v) = \int (u_x^2 + u_y^2 + v_x^2 + v_y^2 + (u \cdot f_x + v \cdot f_y + f_t)^2) \, dx \, dy$

Solution: $u, v = \text{argmin}_{u,v} \{E(u,v)\}$
Optic Flow from Toppoint Movement

Scale Space:

$$(f_0 \ast \phi_s)(x,y) = f(x,y;s)$$
Optic Flow from Toppoint Movement

Scale-space of the image:  \[ f(x, y; s) = (f_0 \ast \phi_s)(x, y) \]
Optic Flow from Toppoint Movement

Scale-space of the image: \( f(x, y; s) = (f_0 * \phi_s)(x, y) \)
Optic Flow from Toppoint Movement

Toppoint definition:

\[
\begin{bmatrix}
    f_x \\
    f_y \\
    \det(H)
\end{bmatrix}
= \begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}
\]

\[\det(H) = f_{xx}f_{yy} - f_{xy}^2\]
Optic Flow from Toppoint Movement

Calculating Toppoint Movement:

\[ f_x = 0 \]
Optic Flow from Toppoint Movement

Calculating Toppoint Movement:

\[ f_x = 0 \]
\[ \frac{d}{dt} (f_x) = 0 \]
Optic Flow from Toppoint Movement

Calculating Toppoint Movement:

\[ f_x = 0 \]
\[ \frac{d}{dt}(f_x) = 0 \]
\[ \partial_t(f_x) + \partial_t x \cdot \partial_x(f_x) + \partial_t y \cdot \partial_y(f_x) + \partial_t s \cdot \partial_s(f_x) = 0 \]
Optic Flow from Toppoint Movement

Calculating Toppoint Movement:

\[ f_x = 0 \]

\[ \frac{d}{dt}(f_x) = 0 \]

\[ \partial_t(f_x) + \partial_t x \cdot \partial_x(f_x) + \partial_t y \cdot \partial_y(f_x) + \partial_t s \cdot \partial_s(f_x) = 0 \]

\[ \partial_t(f_x) + x \cdot \partial_x(f_x) + y \cdot \partial_y(f_x) + s \cdot \partial_s(f_x) = 0 \]

Similar for \( f_y \) and \( \det(H) \rightarrow 3 \) equations with 3 unknowns.
Optic Flow from Toppoint Movement

Find Toppoints

Calculate the toppoint movement
We have the flow at only the toppoint positions:

\[ U_i = \]

We want the flow at all pixels:

\[ u = \]

Therefore, we need a relation between toppoint movement \( U_i \) and the dense flow field \( u \).
Optic Flow from Toppoint Movement

Proposition:

\[ u \times \phi_i = U_i \]
Optic Flow from Toppoint Movement

Formally:

Prior:  Minimize image derivatives

Data term:  Toppoint Movement:  \( u \otimes \phi_i - U_i = 0 \)

Energy Functional:  \( E(u,v) = \int \left( u_x^2 + u_y^2 + v_x^2 + v_y^2 + (u \otimes \phi_i - U_i)^2 + (v \otimes \phi_i - V_i)^2 \right) dxdy \)
Combined Optic Flow

Formally:

Prior: Minimize image derivatives

Data term: Toppoint Movement: $u \otimes \phi_i - U_i = 0$

OFCE: $u \cdot f_x + v \cdot f_y + f_t = 0$

Energy Functional: $E(u,v) = \int \left( u_x^2 + u_y^2 + v_x^2 + v_y^2 + \right.$

$\left. (u \otimes \phi_i - U_i)^2 + (v \otimes \phi_i - V_i)^2 + \right.$

$\left. (u \cdot f_x + v \cdot f_y + f_t)^2 \right) \, dx \, dy$
Combined Optic Flow

To sum things up:

Prior:  
- Maximize smoothness → $u_x^2 + u_y^2 + v_x^2 + v_y^2 + \ldots$
- Toppoint movement → $\alpha_i (u \otimes \phi_i - U_i)^2 + \beta_i (v \otimes \phi_i - V_i)^2 + \ldots$

Data terms:  
- OFCE → $\gamma (u \cdot f_x + v \cdot f_y + f_t)^2$

\[ E(u,v) = \int \]

\[ dx \cdot dy \]
Combined Optic Flow

Implementation:

Energy Functional

→ Variational Calculus

Euler Lagrange Equations

→ Discretisation (with B-Splines)

Linear set of equations

→ Conjugate Gradients

Solution
Results

Ground truth:

Flowfield estimation using our method:

A+B motion?
Results

Error Measure: Angular Error (AE)

\[ u_t' = \]
\[ v_t' = \]
Results

Error Measure: Angular Error (AE)

\[ u^t = \]
\[ v^t = \]
\[ 1 = \]
\[ v^t = \]
Results

Error Measure: Angular Error (AE) = angle between $\mathbf{v}^l$ and $\mathbf{v}^e$. 

$u^l =$

$\mathbf{v}^l =$

$1 =$

$\mathbf{v}^f =$

$\mathbf{v}^e =$
Comparison with other methods

-Average AE:
  -First attempt Traditional Optic Flow: 32.43°
  -Traditional Optic Flow 25 years later: 0.92°
  -First attempt Optic Flow using Toppoints: 17.92°
  -Current best performance our method: 4.80°
### Comparison with other methods

A+B > L ?

<table>
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<th>Method</th>
<th>Average AE</th>
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**Robustness:**

- Basis of Traditional Optic Flow has Aperture Problem and Brightness Constancy
- Basis of our method does not have such problems
Comparison with other methods

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  - Basis of Traditional Optic Flow has Aperture Problem and Brightness Constancy
  - Basis of our method does not have such problems

- Computational Costs:
  - Implementation of Traditional Optic Flow has been perfected over the years, and can now be done real-time
  - Not much attention has been given to the speed-up of our method yet, so it currently takes a few minutes per frame
Future Work

- discontinuity-preserving smoothness
- spatio-temporal smoothness
- replace greyscale conservation by more advanced conservation term
- search for other anchor points besides toppoints
- change relation between anchor point movement and flow field
- make weights dependent on reliability measure
Thank you for your attention.

Any questions?